Symmetry-Based Quantum Circuit Mapping

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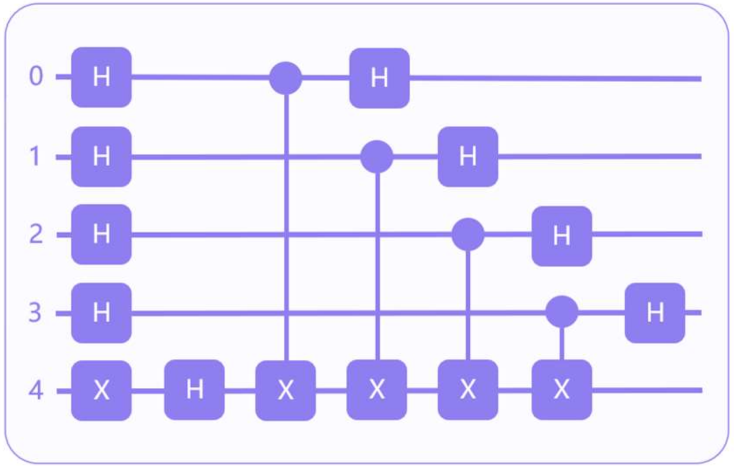
Sep 21, 2024

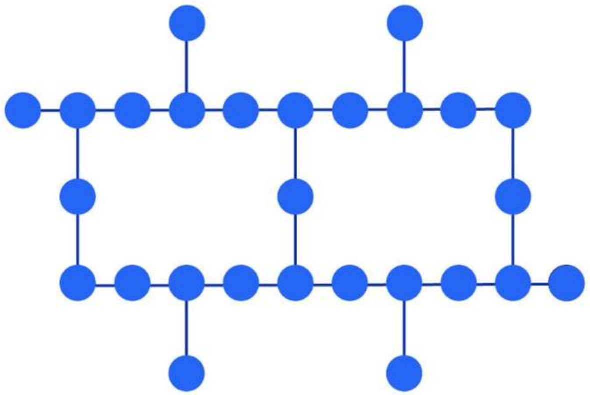
Based on Phys. Rev. Applied 22, 024029 (ArXiv: 2310.18026)

Quantum circuit compilation and mapping

Can we implement the following quantum circuit with IBM Falcon?



Example quantum circuit



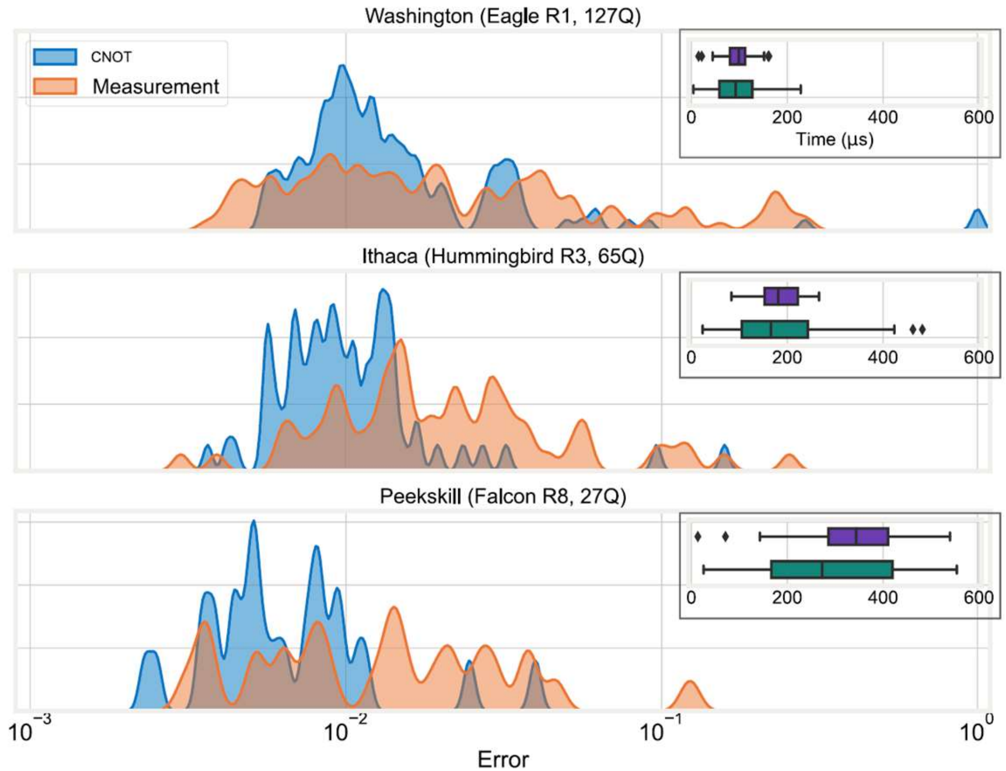
Coupling graph of IBM Falcon[1]

Not executable due to limited qubit connectivity!

Quantum circuit compilation: transforming a quantum circuit to make it physically executable on a quantum hardware with maximal circuit fidelity.
Qubit mapping: determine which physical qubits will be used for executing a logical quantum circuit.

[1] https://www.ibm.com/quantum/blog/heavy-hex-lattice

System variability in quantum computing systems

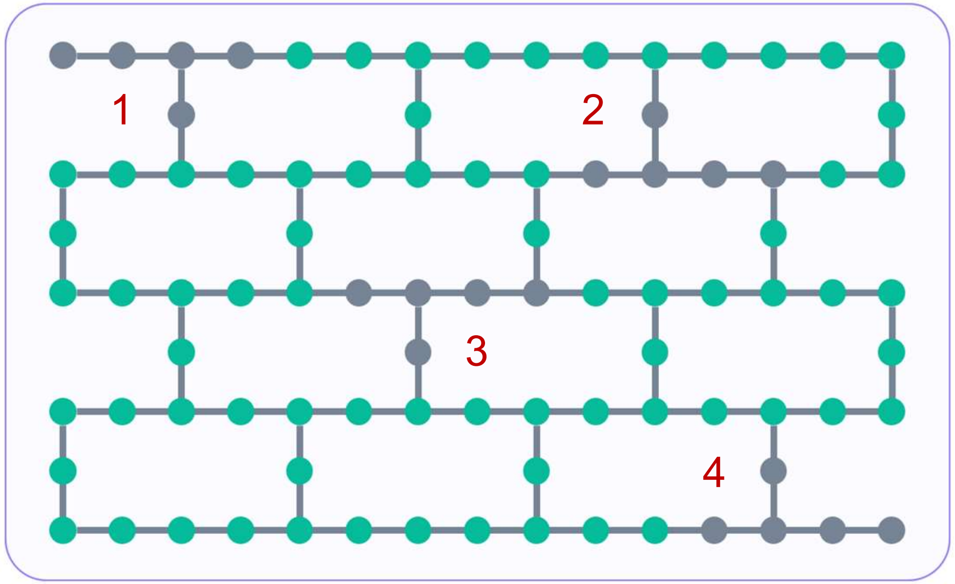


Distribution of gate error and measurement error in three IBM quantum systems[1]

- In a real quantum system, gate error and measurement error may vary significantly with qubits.
- Taking this system variability into account can help find the optimal circuit mapping.

[1] PRX Quantum 4, 010327 (2023)

Suppress system variability with circuit remapping



Some subset of qubits (shown in grey) have equivalent connectivity and offer alternative circuit mappings

Paul D. Nation and Matthew Treinish proposes remapping for improving compilation process:

1. Pre-compilation with existing routine → e.g. circuit mapping to qubits in subset 1
2. Identify qubit subsets with equivalent connectivity, such as subsets 2, 3, and 4
3. Find one subset that maximizes circuit fidelity for implementing the desired quantum circuit

[1] PRX Quantum 4, 010327 (2023)

Subgraph matching problem

The quantum circuit remapping involves finding subsets of qubits having the same connectivity. Below we rephrase this problem using graph theory.

A graph is an ordered pair $G = (V, E)$ comprising:
- V, a set of vertices
- $E \subset \{(x, y) | x, y \in V \text{ and } x \neq y\}$

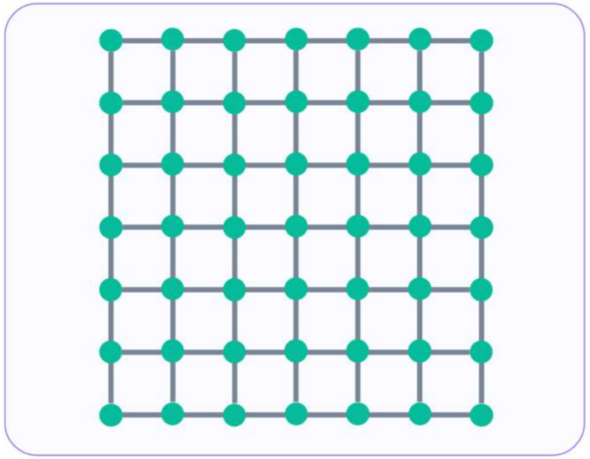
An isomorphism of graphs G and H is a bijection between the vertex sets of G and H $f: V(G) \to V(H)$ such that any two vertices u and v of G are adjacent in G if and only if $f(u)$ and $f(v)$ are adjacent in H.

Subgraph matching problem: Given a graph G and a graph T, what are all subgraphs in G that are isomorphic to T?

(Stephen A. Cook, 1971) Subgraph matching problem is nondeterministic polynomial (NP) time complete in general.

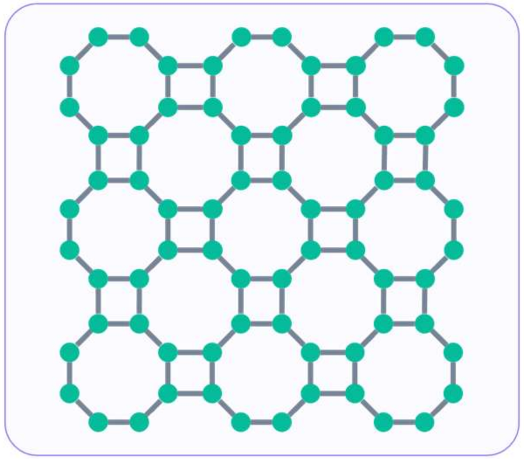
Common quantum hardware structures and their symmetries

Qubit connectivity of some representative quantum computing systems



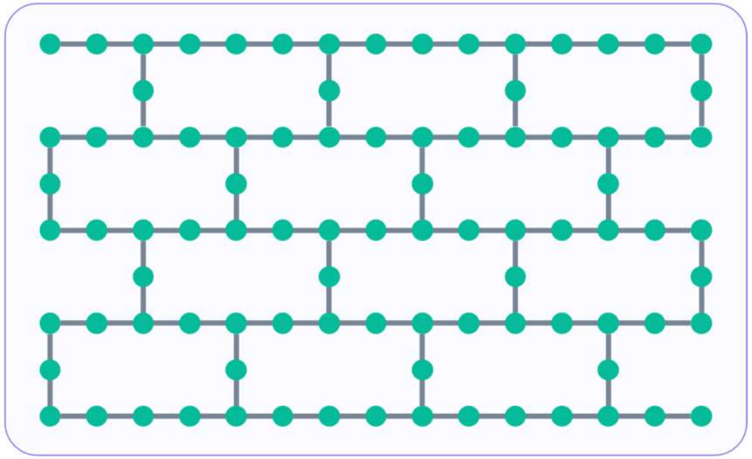
(a) 2D grid

Example: Google Sycamore



(b) Octagonal

Example: Rigetti



(c) Heavy-hex

Example: IBM Falcon

The hardware structure exhibits translation symmetry, rotation symmetry, inversion symmetray…

Can we use these symmetries as priori knowledge to facilitate the subgraph matching and circuit remapping?

Symmetry-based quantum circuit mapping: overview

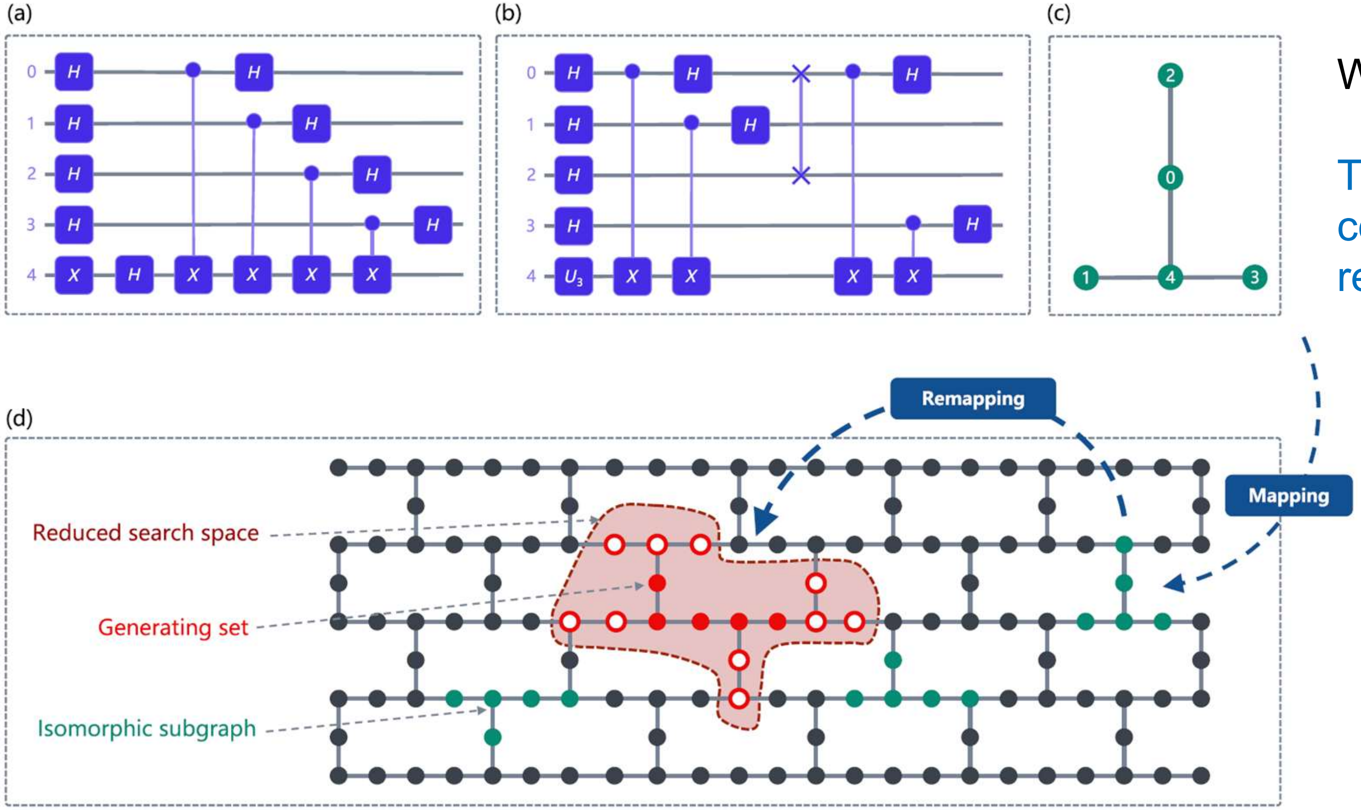


Illustration of quantum circuit compilation with remapping process

We have shown that:

The circuit remapping process can be confined to a reduced search space, greatly reducing computing load.

Symmetry-based quantum circuit mapping: theorem

Definition Given a graph $G = (V, E)$ and a group of automorphisms H, a subset of vertices S s.t. $V = \{f(v) | f \in H, v \in S\}$ is a generating set of G with respect to the group of automorphisms H.

Theorem Let T be a graph and S be a generating set of T with respect to a group of automorphisms $F.$ Let G be a subgraph of T with radius r. Then for any subgraph G'' of T that is isomorphic to G, there exists an automorphisms $f \in F$ and subgraph $G' \subset T$ such that $G'' = f(G')$, with G' isomorphic to G, $V(G') \subset N_T^r(S)$.

Symmetry-based quantum circuit mapping: algorithm

ALGORITHM 1. Symmetry-based subgraph matching

Input : A pattern graph G, a target graph T, a group of automorphisms F of the target graph and the associated generating set S.

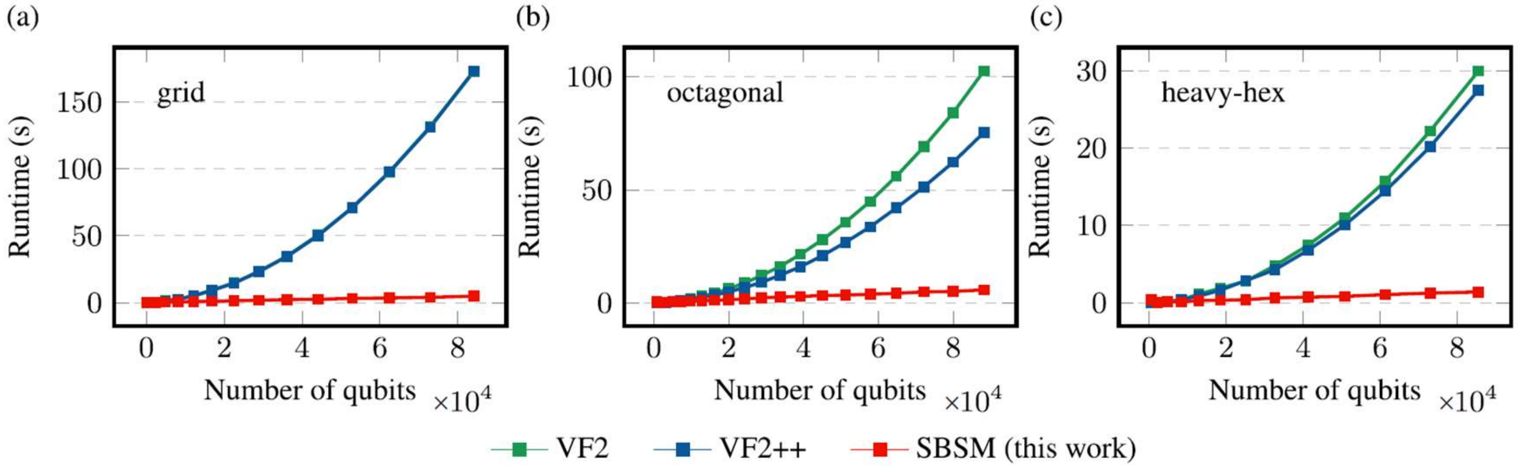
Output: All subgraph isomorphisms of G in T.

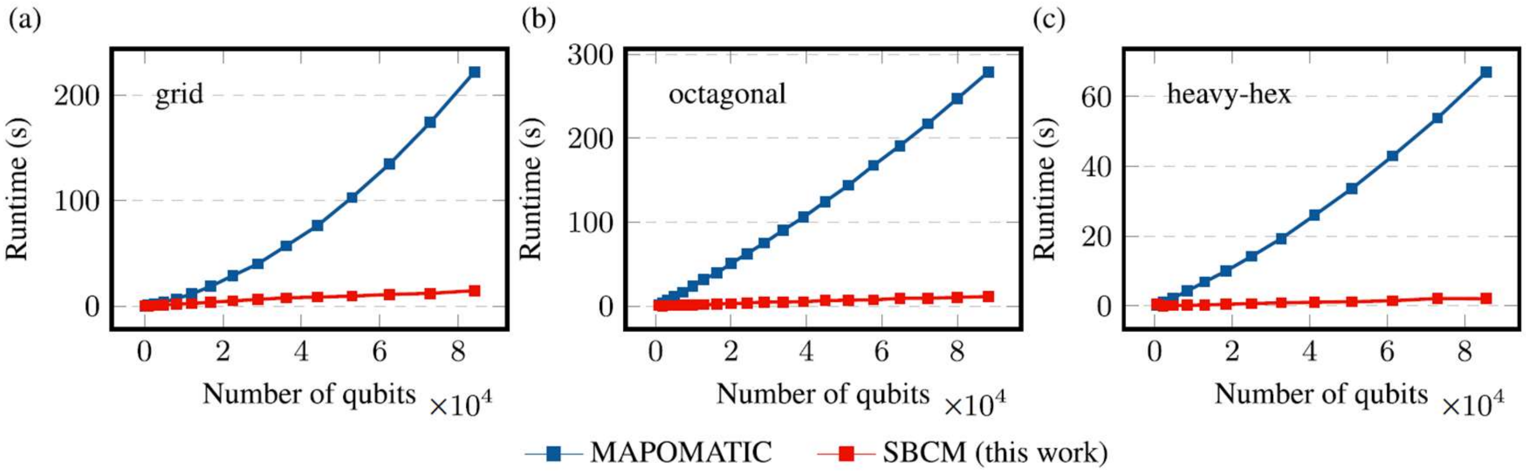
Let r be the radius of the pattern graph G;
Let $N_T^r(S)$ be the r-th order neighborhood of S;
Let R be the induced subgraph of $N_T^r(S)$ in T;
Obtain the set of all isomorphic graphs of G within R and denote this set as H_0;
Let H be an empty list;
for $G' \in H_0$ **do**
    Let $M = \{\tilde{f}(G') : f \in F, \tilde{f}(G') \subseteq T\}$;
    Append M to H;
end
Return H;

We have shown that, in many practical cases, this algorithm has a time complexity of $O(n)$ with n the number of qubits in the hardware, which is optimal for the subgraph matching problem.

Algorithm	Time complexity
VF2	$O(n^2) \sim O(n!\, n)$
SBSM (this work)	$O(n)$

Symmetry-based quantum circuit mapping: benchmark results





Summary

- There are significant fluctuations in gate error and measurement error across existing quantum computing systems.

- The impact of such fluctuations can be suppressed by quantum circuit remapping process.

- Quantum circuit remapping involves solving the subgraph matching problem, which is NP-complete.

- We developed an algorithm, using symmetries in hardware as priori knowledge, to significantly reduce computing load of quantum circuit remapping. This algorithm features the optimal time complexity in many practical cases.