Online Locality Meets Distributed Quantum Computing

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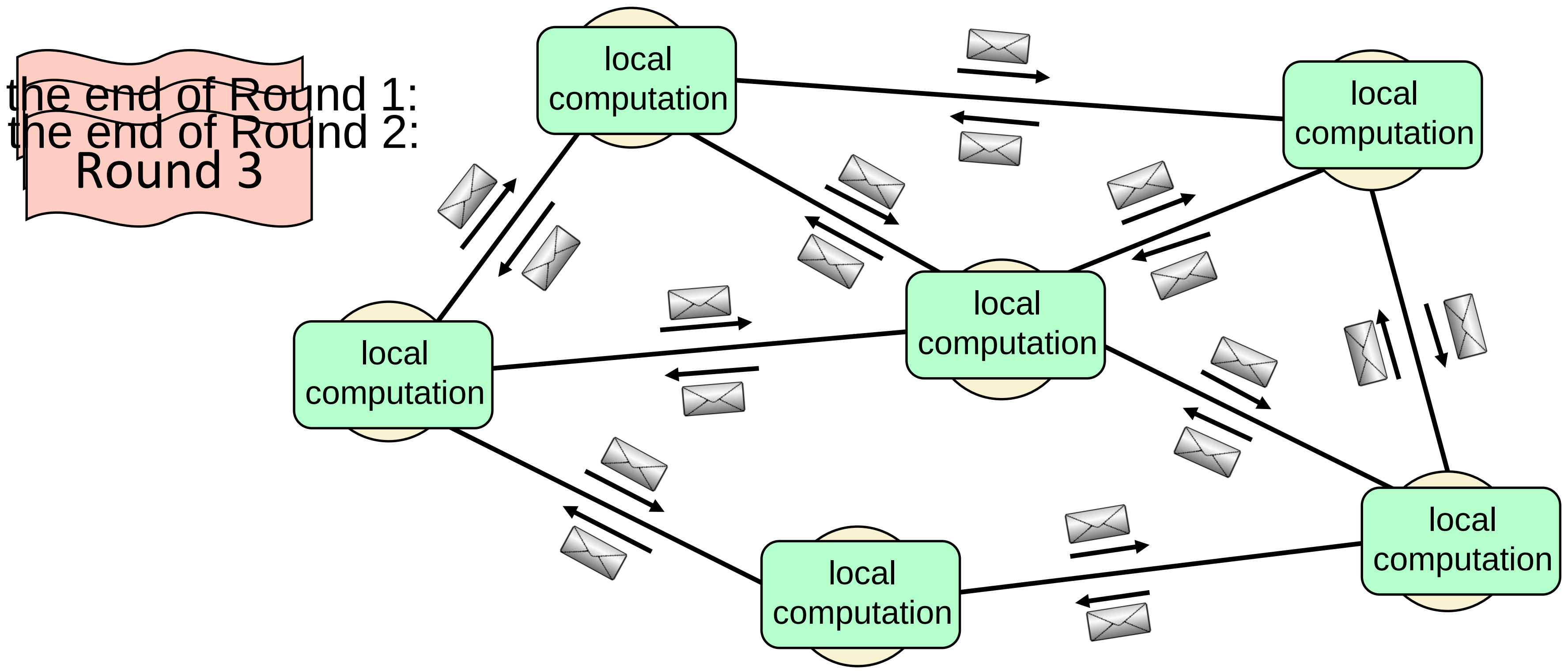
Classical Distributed Computing

- ✓ network G = (V,E) of n nodes (all nodes have distinct identifiers)
- ✓ each node initially knows nothing about the topology of the graph
- ✓ synchronous communication between adjacent nodes:

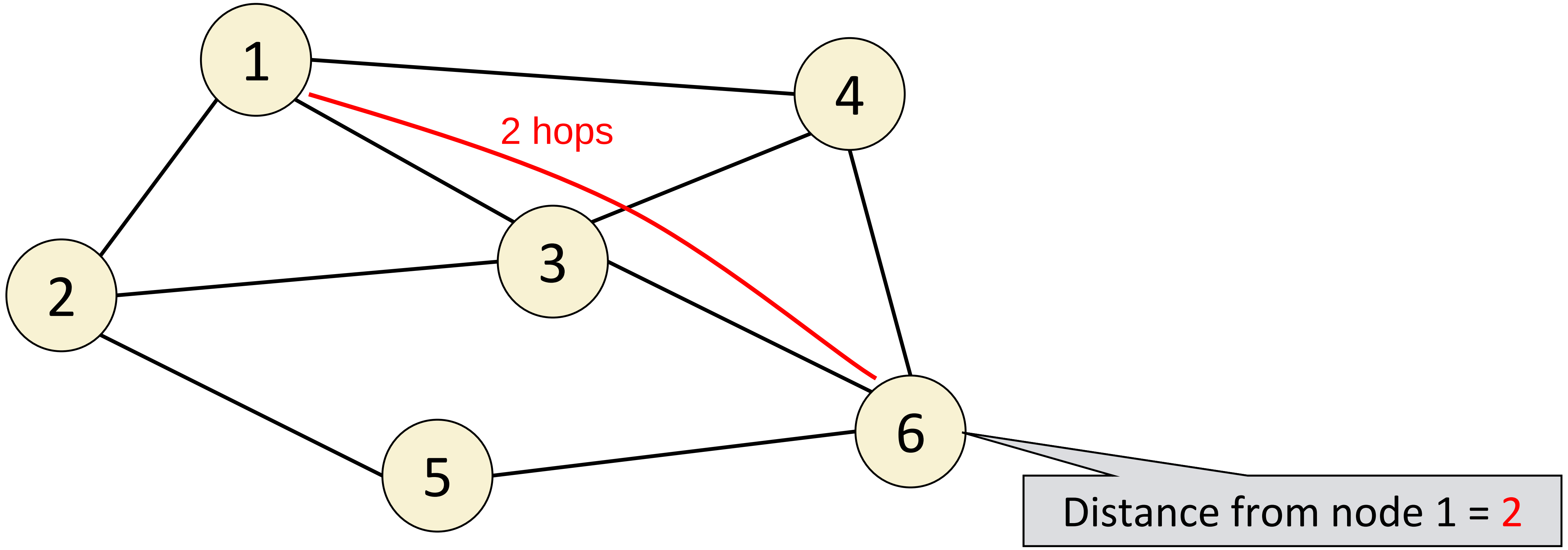
  one message through each edge per round (in each direction)

Complexity: the <u>number of rounds</u> used

at the end of Round 1:
at the end of Round 2:



Example of Classical Distributed Algorithm: Computing Distances



Example of Classical Distributed Algorithm: Computing Distances

Round 1 Node 1 sends a message to its neighbors

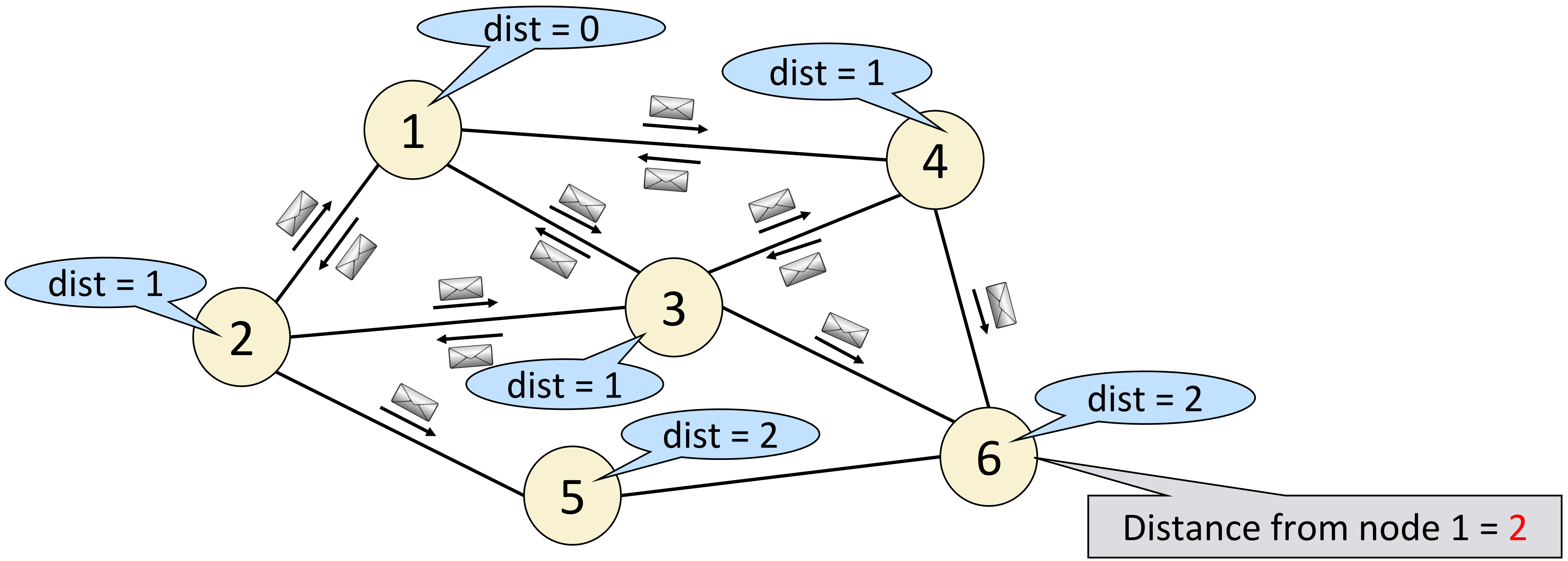
at the end of Round 1: each node updates its distance

(nodes that received a message at Round 1 set “dist = 1”)

Round 2 nodes tell new knowledge to neighbors

at the end of Round 2: each node updates its distance

(nodes that received a message for the first time at Round 2 set “dist = 2”)



Classical Distributed Computing

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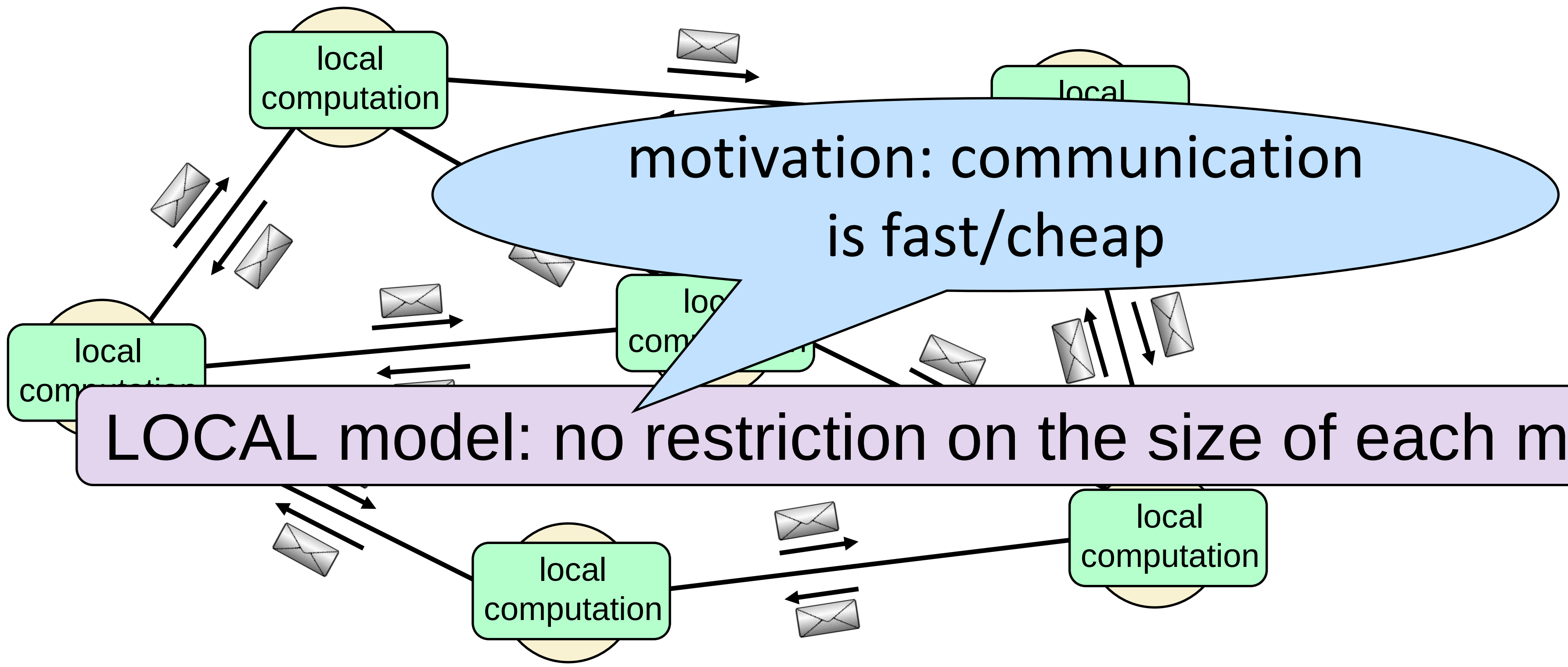
Complexity: the number of rounds used

what size?

CONGEST model: only 1 bit per message



LOCAL model: no restriction on the size of each message

Quantum Distributed Computing

Quantum distributed computing

Now qubits can be sent instead of bits

(no prior entanglement between nodes)

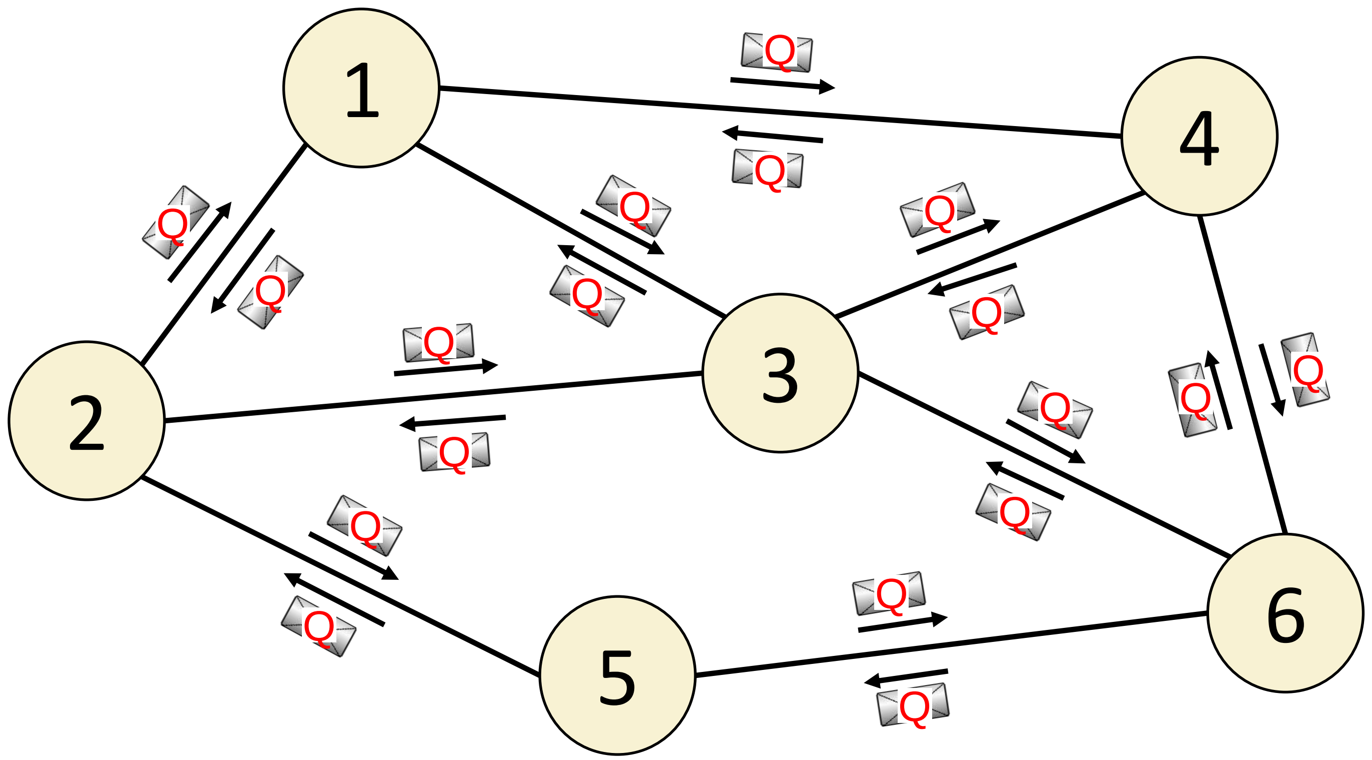
CONGEST model: only 1 qubit per message

✓ related to (quantum) communication complexity

✓ several known examples of quantum advantage (polynomial speedups) obtained recently

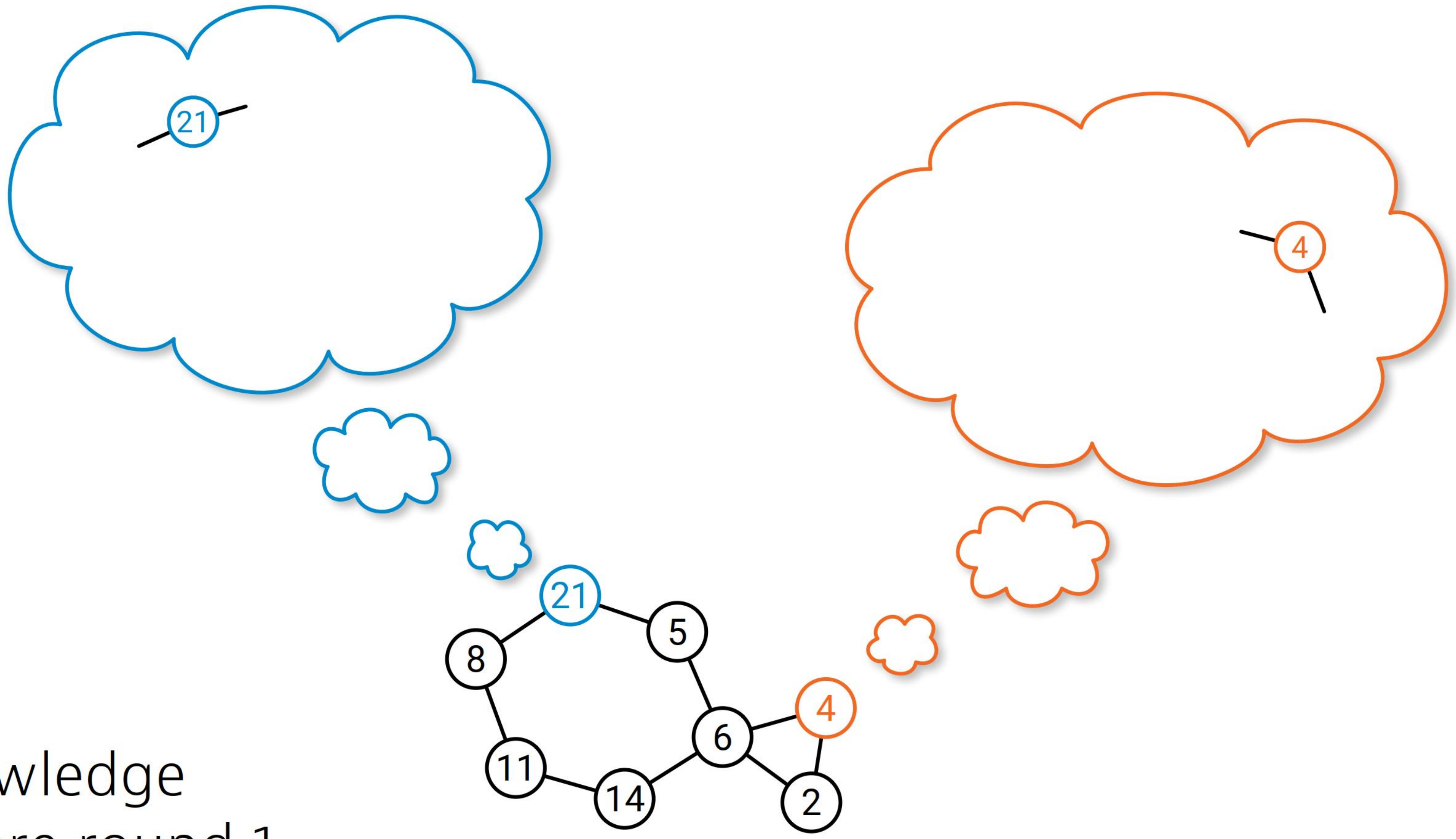
LOCAL model: no restriction on the size of each quantum message

✓ very few results



LOCAL Model: Classical (Deterministic) Algorithms

Typically, we consider a bounded-degree graph, nodes do not receive any input (except their IDs), and we want to solve a problem related of the whole graph (e.g., compute a graph coloring)



Knowledge before round 1

LOCAL Model: Classical (Deterministic) Algorithms

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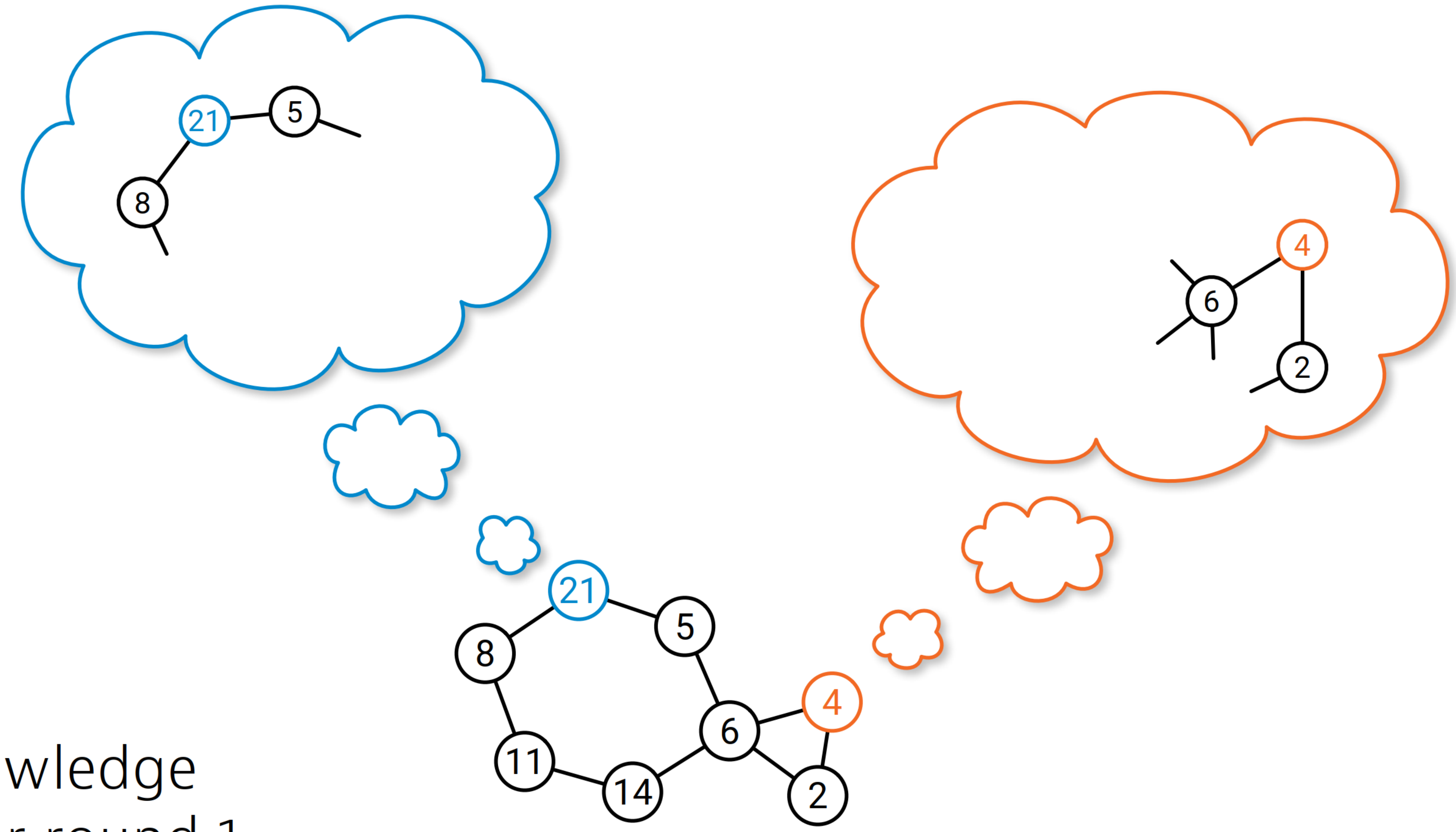

Knowledge
after round 1

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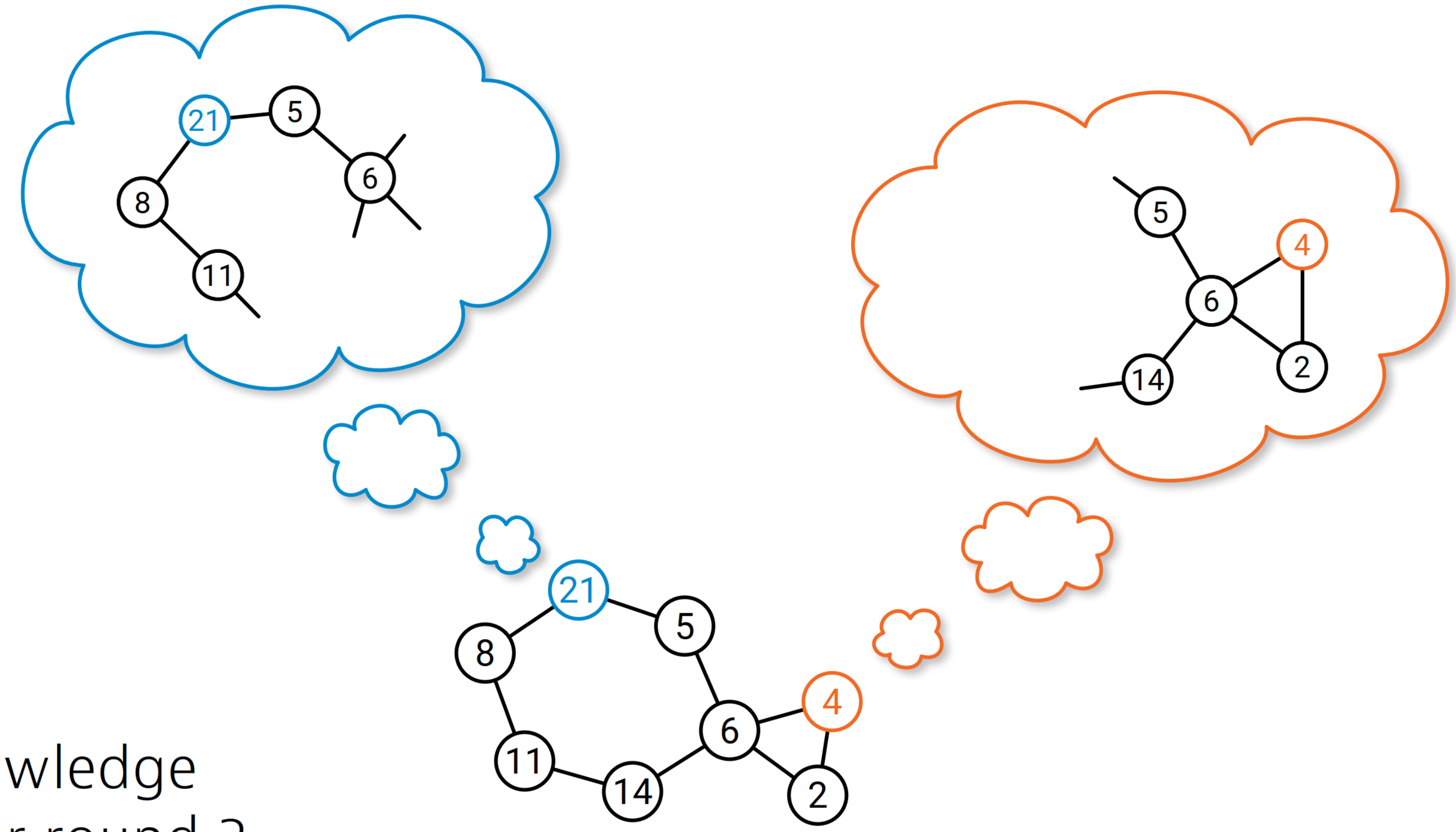
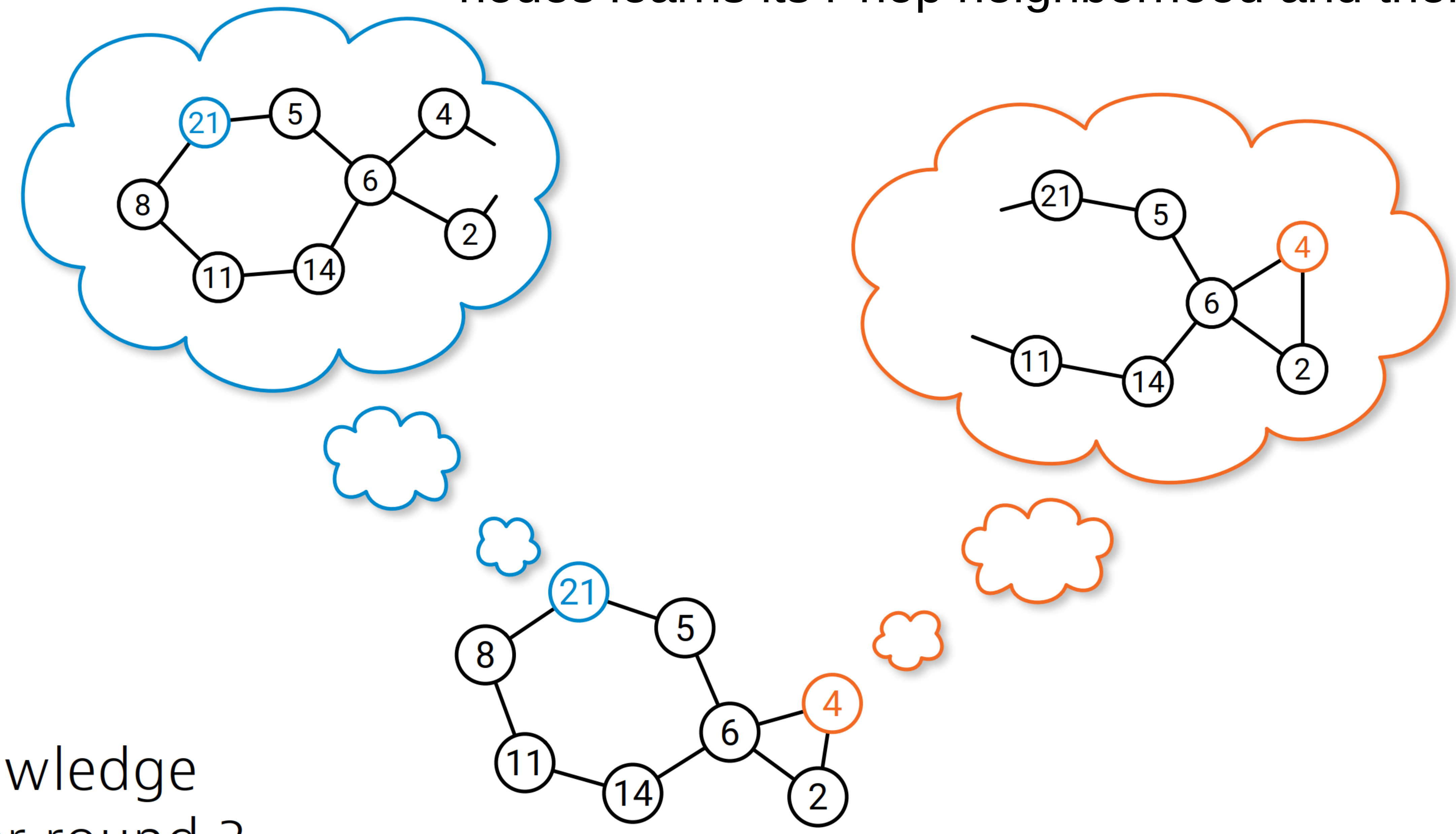


Knowledge
after round 2

LOCAL Model: Classical (Deterministic) Algorithms

Typically, we consider a bounded-degree graph, nodes do not receive any input (except their IDs), and we want to solve a problem related of the whole graph (e.g., compute a graph coloring)

This is the <u>optimal strategy</u>: without loss of generality, we can assume that in an r-round algorithm, the nodes learns its r-hop neighborhood and then compute their output locally



Knowledge
after round 3

LOCAL Model: Classical (Deterministic) Algorithms

number of communication rounds

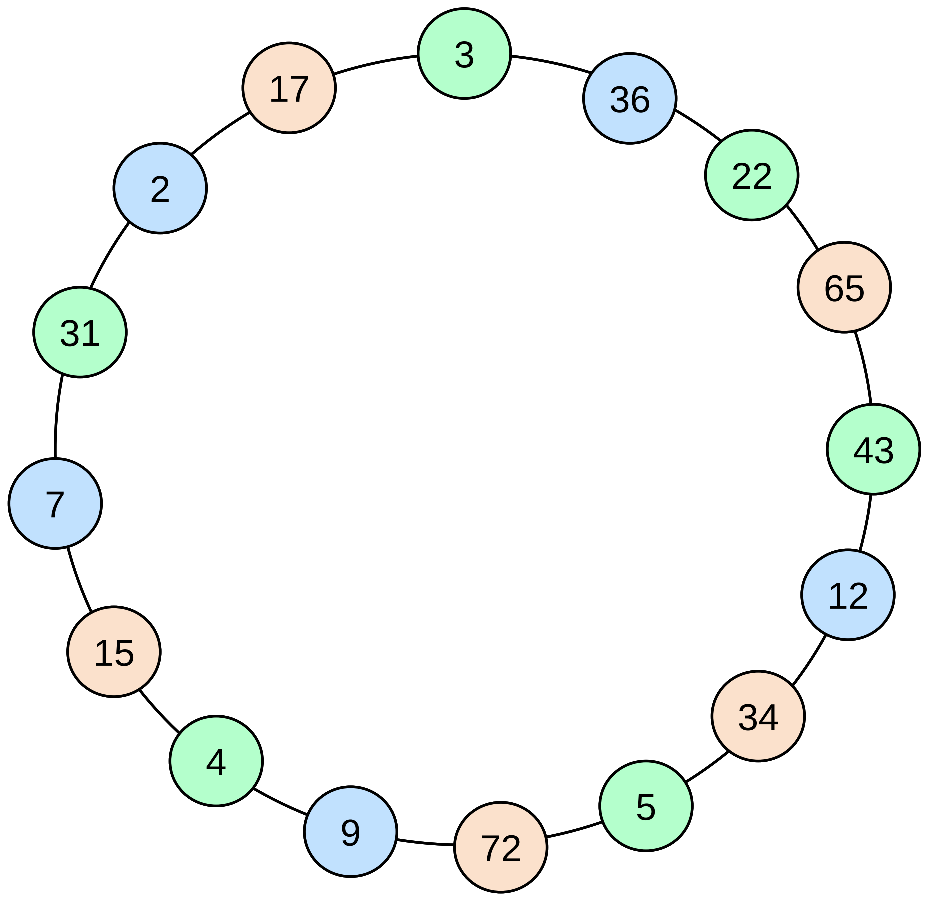
=

how far do you need to see

⇨ tight bounds can be obtained on the classical complexity of many problems

Basic Problem: 3-Coloring on Rings

Any ring has a 3-coloring (i.e., a node-coloring where neighbors have distinct colors)

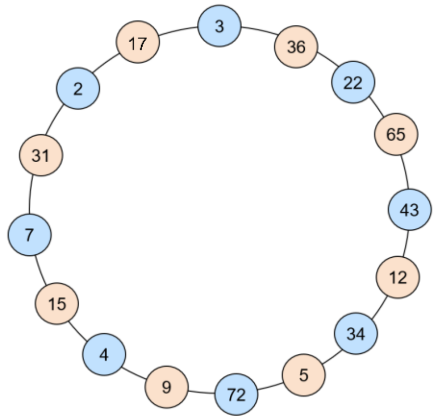


n: number of nodes

In the deterministic LOCAL model, a 3-coloring of a ring can be computed in Θ(log*n) rounds [Cole and Vishkin 1986] [Linial 1992]

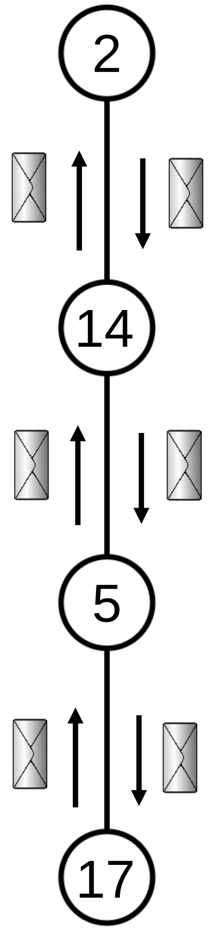
log*n: number of times the log function must be iteratively applied before the result is less than or equal to 1
(example: $\log^*(2^{65536}) = 5$)

Remark: if n is even then there exists a 2-coloring, but computing it requires Θ(n) rounds



LOCAL Model: Quantum Algorithms

1 quantum message (unbounded length) between adjacent nodes per round



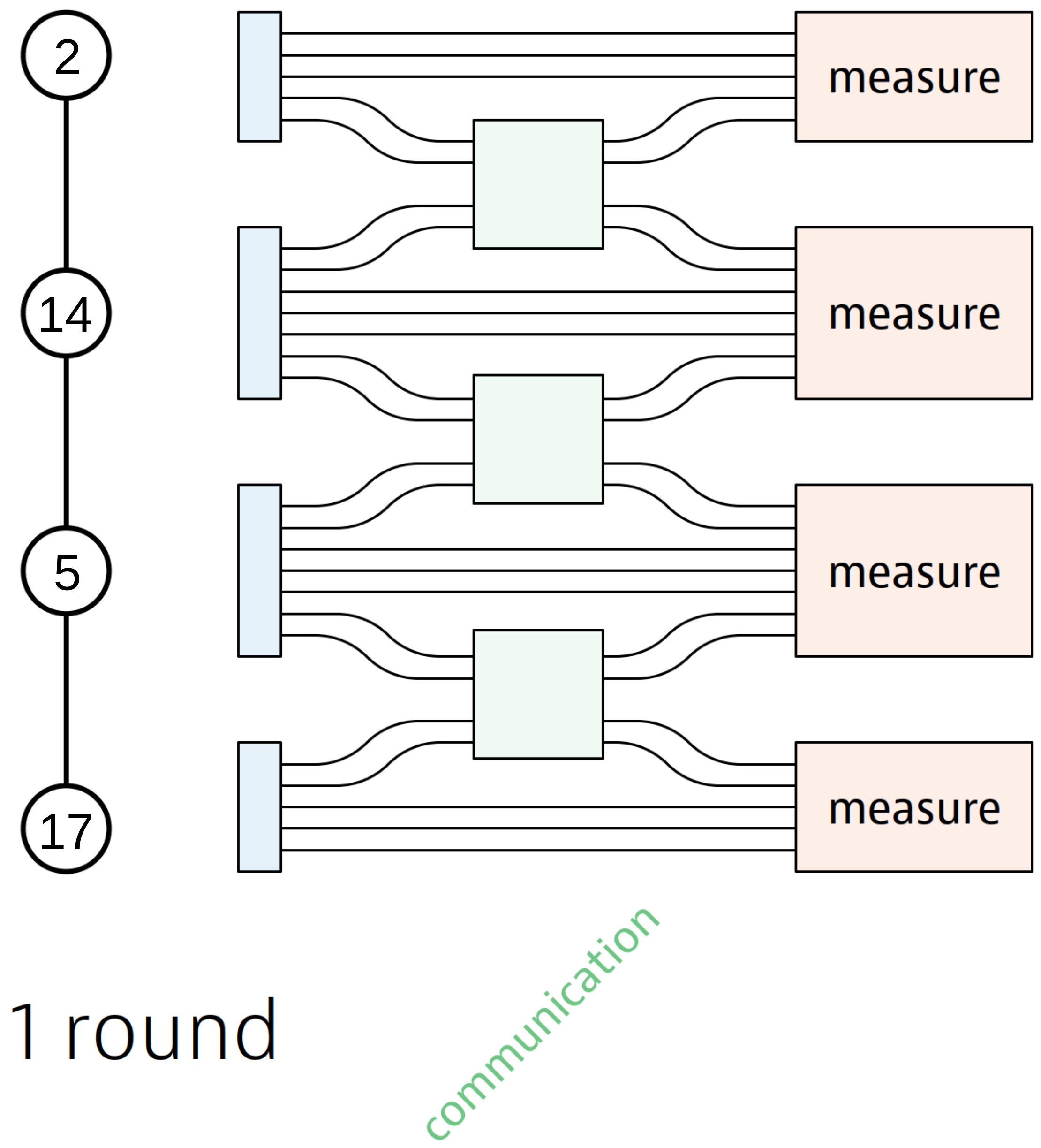
LOCAL Model: Quantum Algorithms

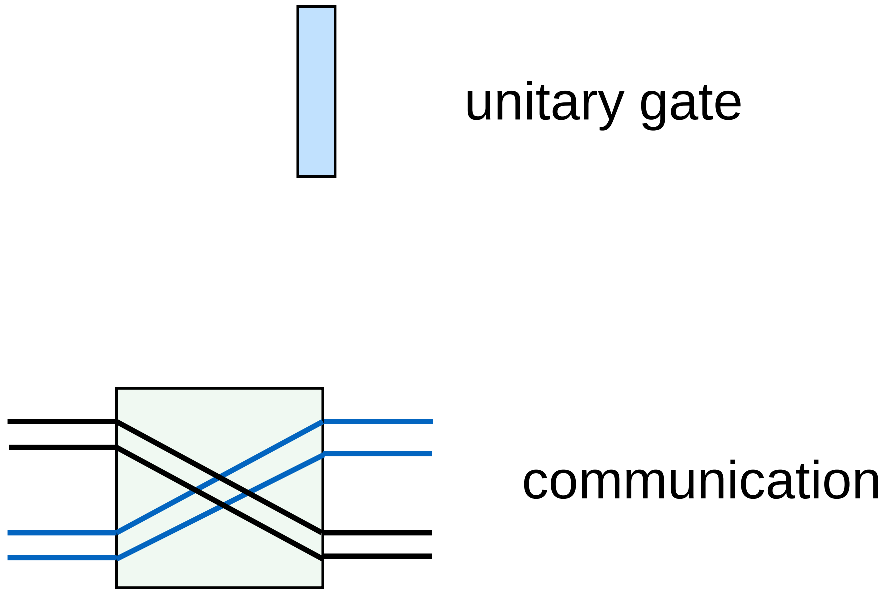
1 quantum message (unbounded length) between adjacent nodes per round

0 rounds

LOCAL Model: Quantum Algorithms

1 quantum message (unbounded length) between adjacent nodes per round





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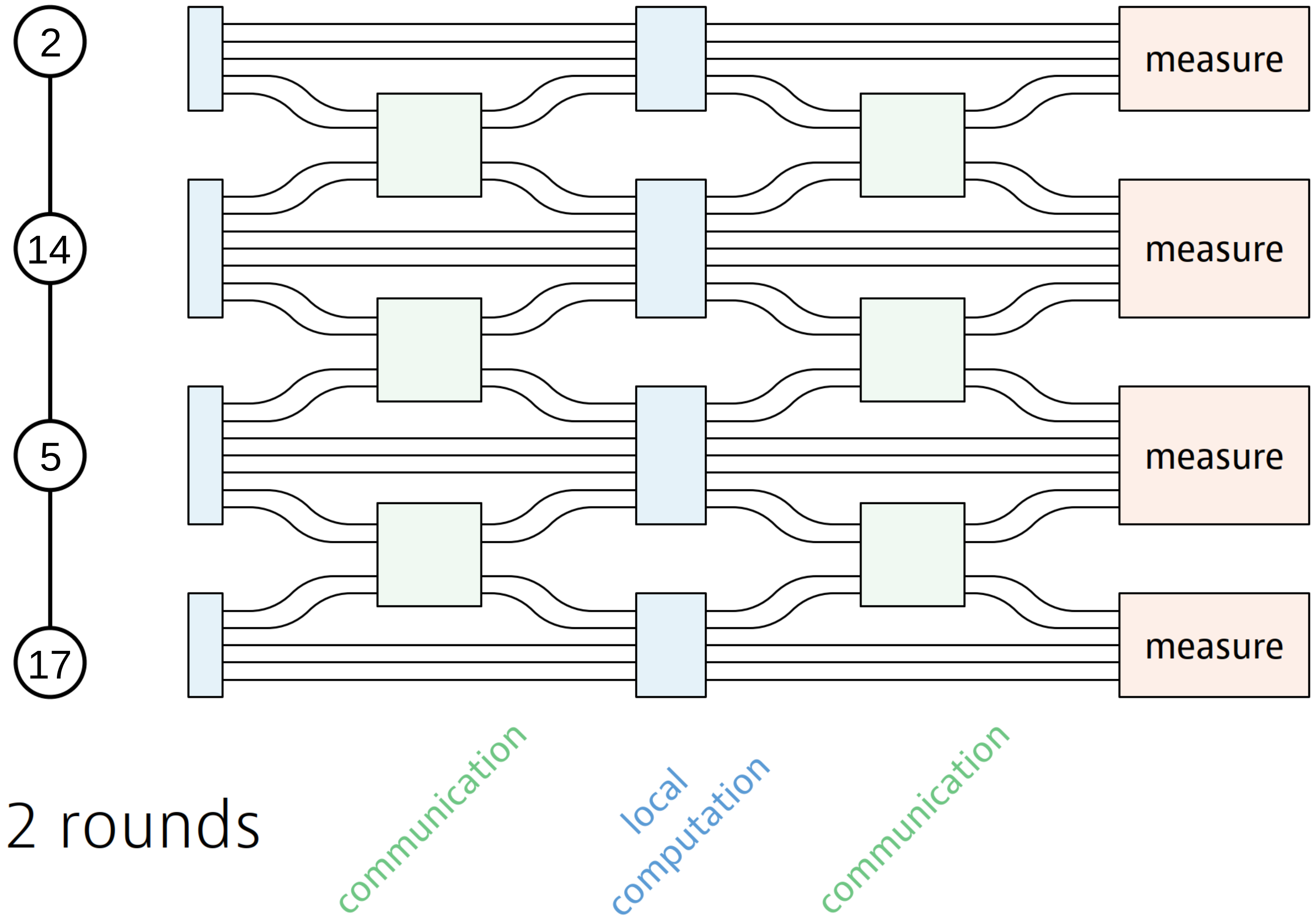

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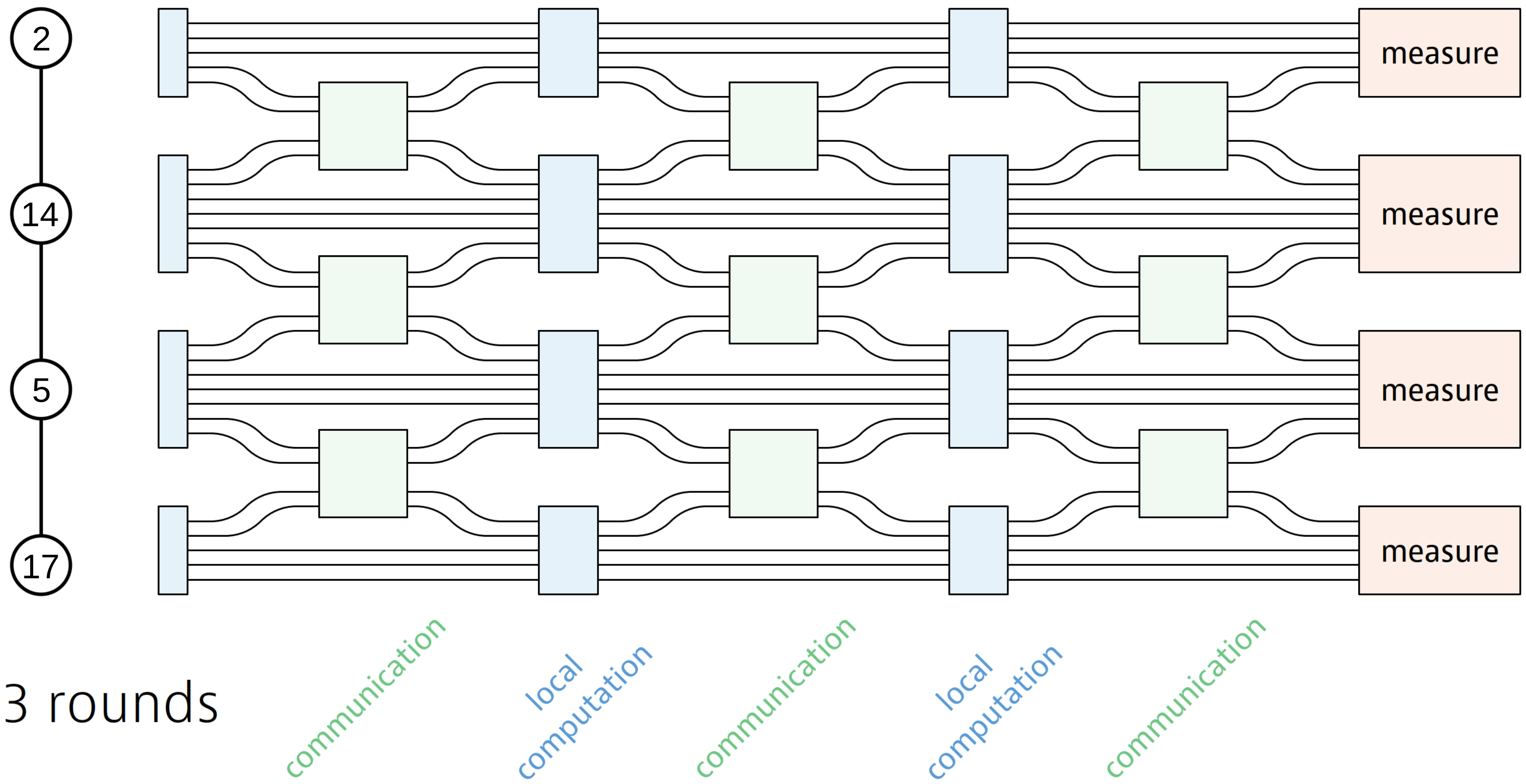


Quantum Advantage in the LOCAL Model?

Is there any problem for which we can show a distributed quantum advantage?

YES!

[Gavoille, Kosowski, Markiewicz DISC'09] ⇨ There is a computational problem that can be solved in 1 round in the quantum LOCAL model but requires 2 rounds classically.

sampling from the outcome of a quantum circuit that measures a graph state in a random basis

[LG, Nishimura, Rosmanis STACS'19] ⇨ There is a computational problem that can be solved in $O(1)$ rounds in the quantum LOCAL model but requires $\Theta(n)$ rounds classically.

two weaknesses:

- ✓ the computational task is not useful
- ✓ the solutions are not efficiently checkable (checking if the solution is correct requires $\Theta(n)$ rounds)

remark: situation similar to "quantum supremacy" with quantum circuits

Better Quantum Advantage in the LOCAL Model?

Is there any problem that someone actually cares about for which we can show distributed quantum advantage?

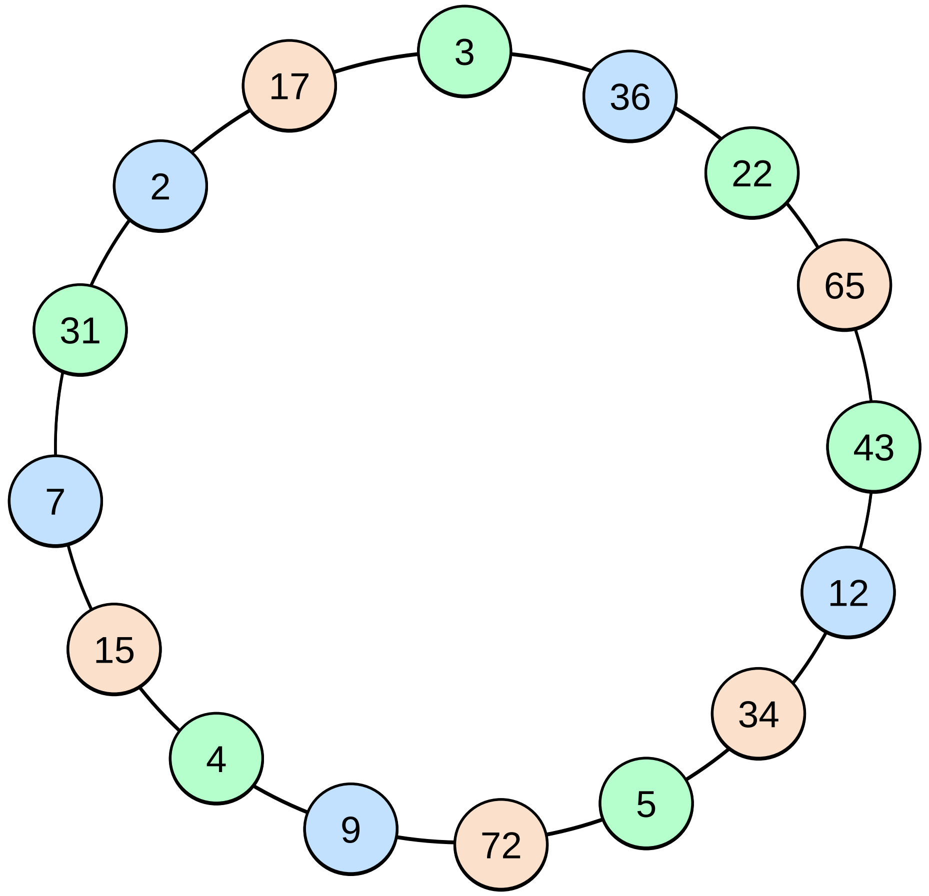
Is there any problem that is efficiently checkable for which we can show distributed quantum advantage?

Nobody knows!

Basic Problem: 3-Coloring on Rings

Any ring has a 3-coloring (i.e., a node coloring where neighbors have distinct colors)

This is an efficiently checkable problem: each node only needs to check (in 1 round) if its color is distinct from the colors of its two neighbors



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log*n: number of times the log function must be iteratively applied before the result is less than or equal to 1

(example: $\log^*(2^{65536}) = 5$)

Fundamental question:
Can we do better (e.g., O(1) rounds) in the quantum setting?

3-coloring rings
Classical: $\Theta(\log^* n)$
Quantum: ???

Already asked in the early papers [Gavoille, Kosowski, Markiewicz DISC'09] [Arfaoui, Fraigniaud 2014] on quantum distributed computing

Better Quantum Advantage in the LOCAL Model?

locally checkable

Is there any problem that is ~~efficiently checkable~~ for which we can show distributed quantum advantage?

Nobody knows!

3-coloring rings
Classical: $\Theta(\log^* n)$
Quantum: ???

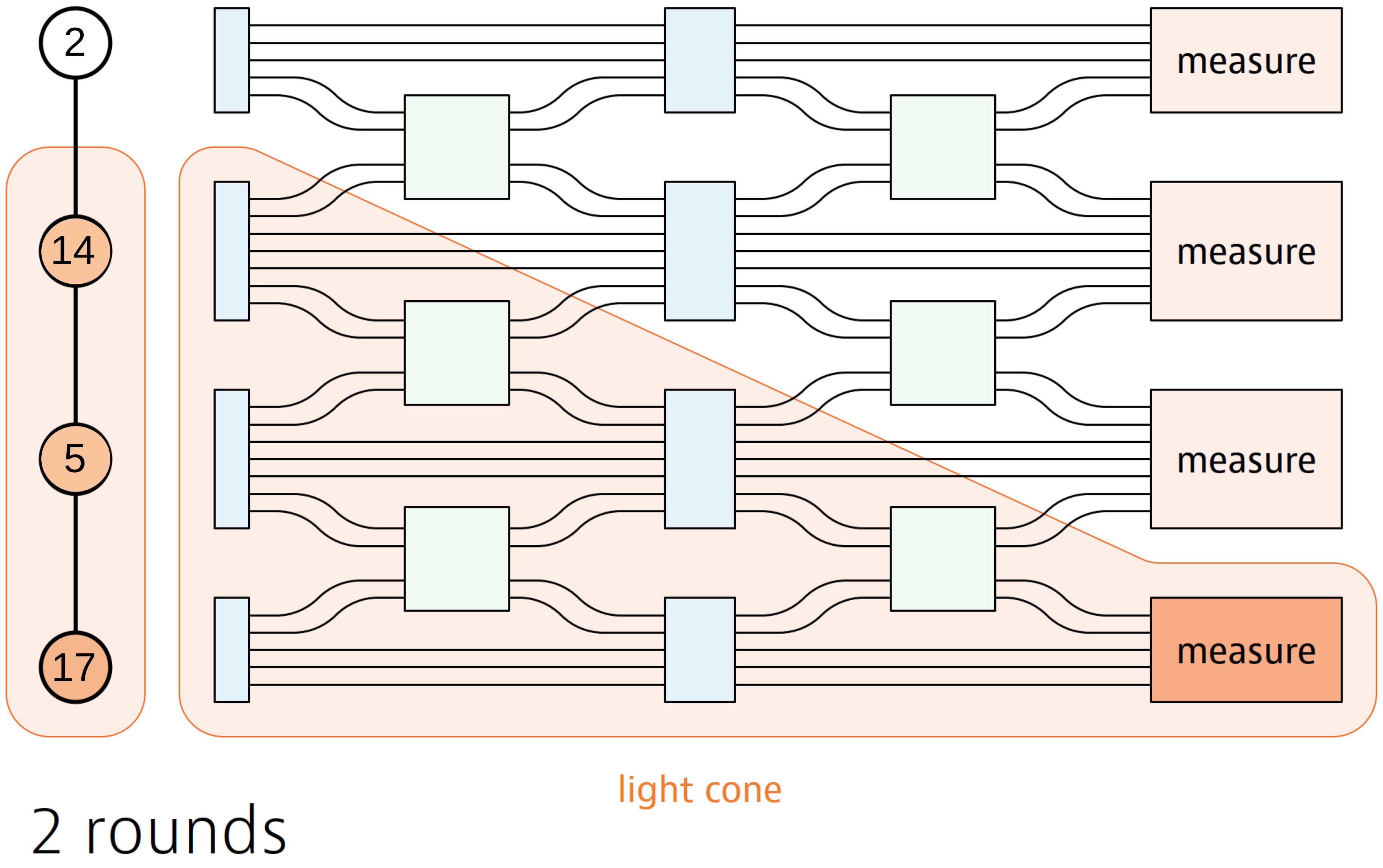
Essentially all problems studied in the literature have the same property as 3-coloring: they are locally checkable (i.e., the nodes can check if a solution is valid in O(1) rounds)

Natural conjecture:

for all locally-checkable problems, there is no quantum advantage in the LOCAL model

Causality

Quantum distributed algorithms satisfy causality



this output only depends on the inputs within the light cone

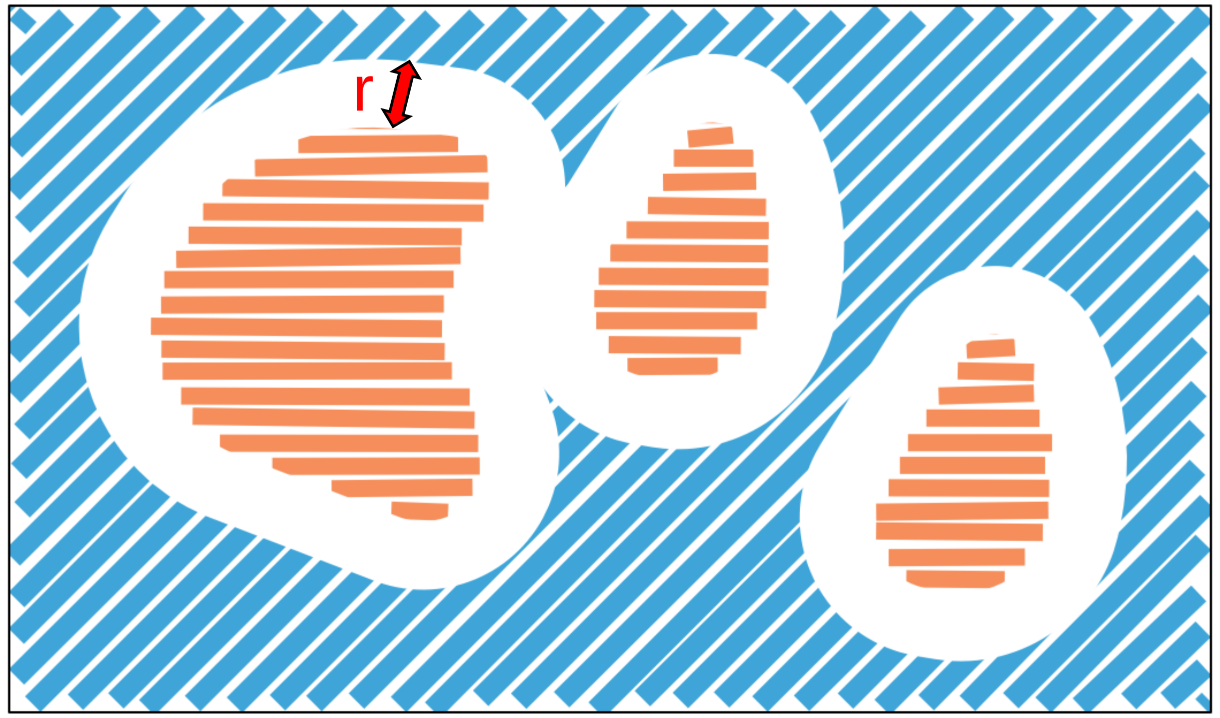
Non-Signaling “~~Algorithms~~” Distributions

Key idea: define a model so that it can do anything except violating causality

Definition (r-hop non-signaling distribution):

- ✓ fix any **set of nodes X**
- ✓ changes in the input **more than r hops away** from **X** do not influence the output distribution of **X**

[Gavoille, Kosowski, Markiewicz DISC’09][Arfaoui, Fraignaud 2014]



Three Models

how far do you need to see?

Classical (deterministic or randomized) distributed algorithms

↓

Quantum distributed algorithms

???

↓

Non-signaling “algorithms” (non-signaling distributions)

classical probability theory

Conjecture: for all locally-checkable problems, there is no quantum advantage in the LOCAL model

Natural approach to prove the conjecture:

Show that for any locally-checkable problem, non-signaling “algorithms” are not more powerful than classical algorithms (this would imply “classical = quantum = non-signaling”)

Our Main Result

Conjecture: for all locally-checkable problems, there is no quantum advantage in the LOCAL model

Natural approach to prove the conjecture:

Show that for any locally-checkable problem, non-signaling "algorithms" are not more powerful than classical algorithms (this would imply "classical = quantum = non-signaling")

✓ Proved in [Gavoille, Kosowski, Markiewicz DISC'09] for 2-coloring in rings

✓ Proved in [Coiteux-Roy et al., STOC'24, TQC'24] for graph coloring in arbitrary bipartite graphs

yesterday's talk

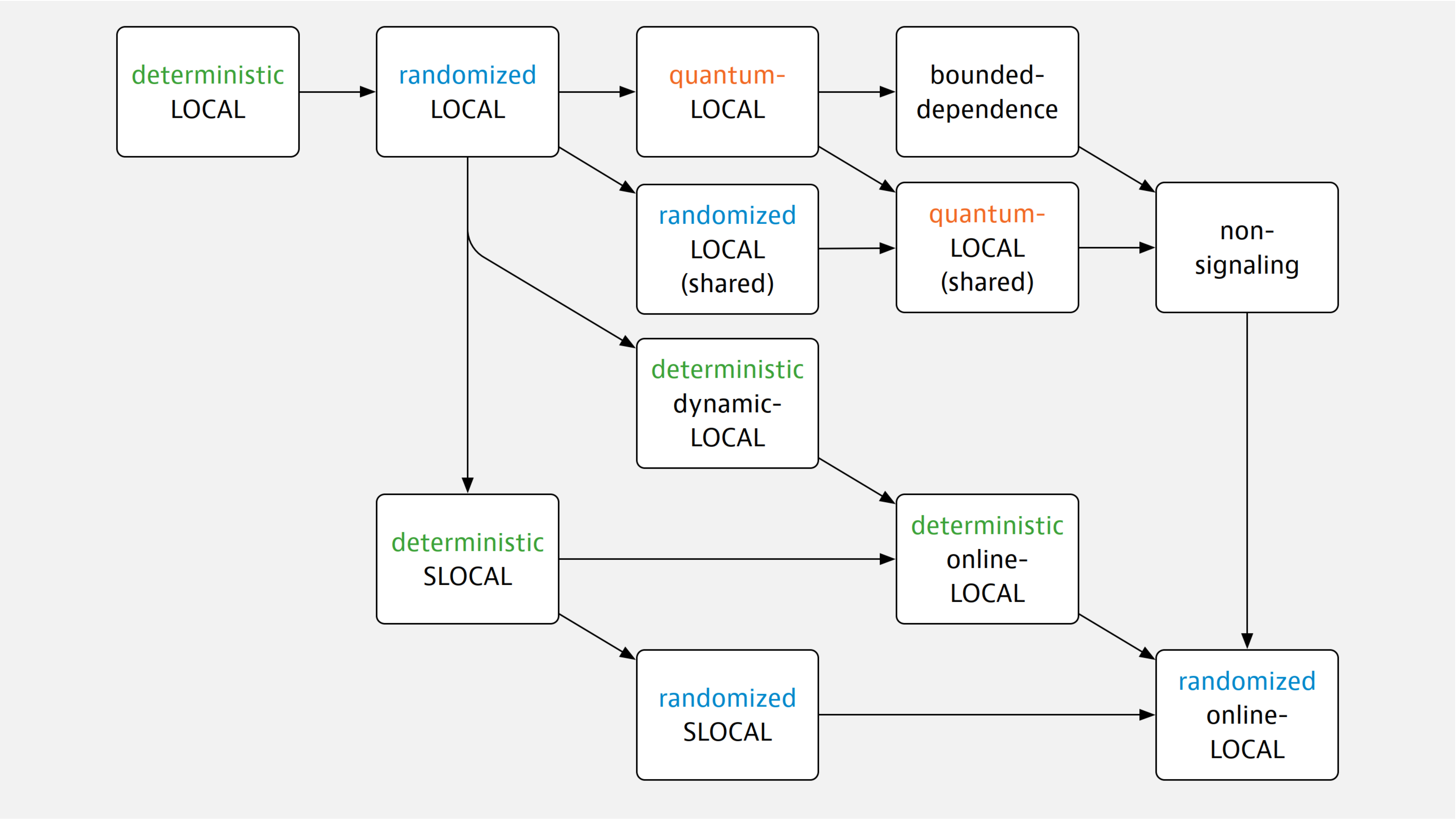
Our main result (informal)

There exist many classes of locally-checkable problems for which non-signaling "algorithms" are more powerful than classical algorithms

⇒ this natural approach doesn't work

(positive interpretation: there might be a quantum advantage!)

All the Models Discussed in our Paper



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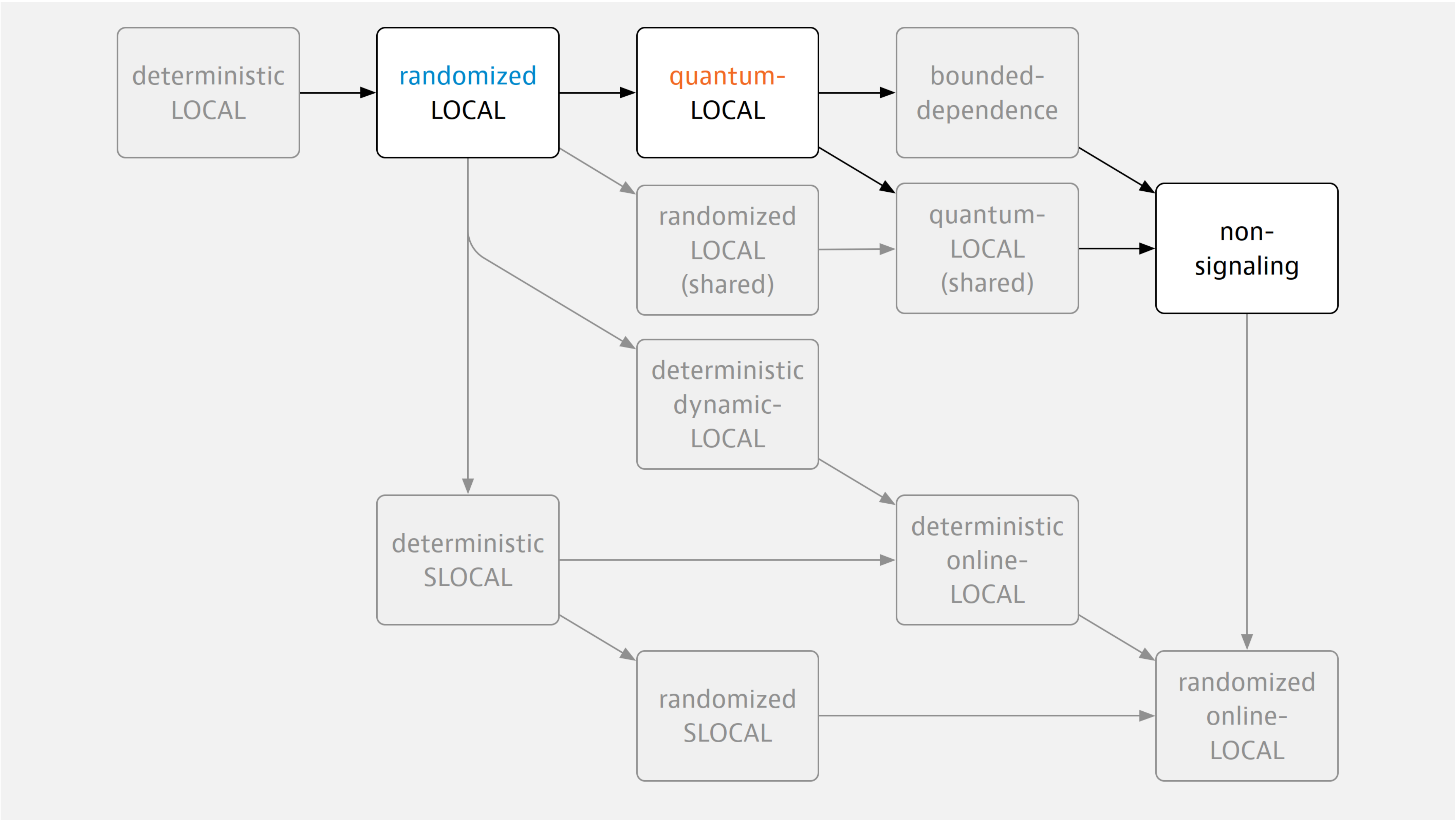
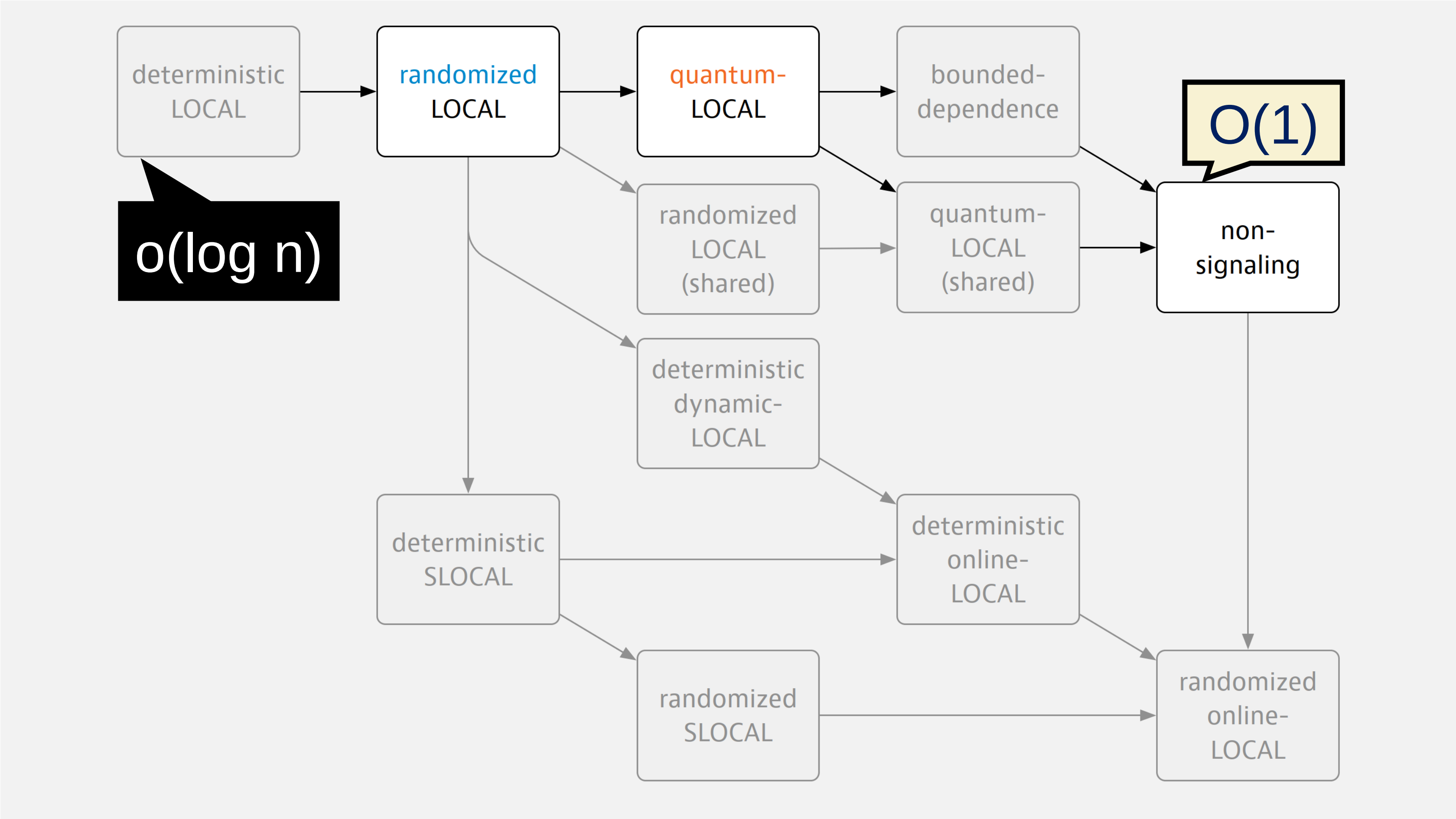


Main Result

Theorem 1

Any locally-checkable problem that can be solved in o(log n) rounds in the deterministic LOCAL model can be solved in O(1) rounds in the non-signaling model.

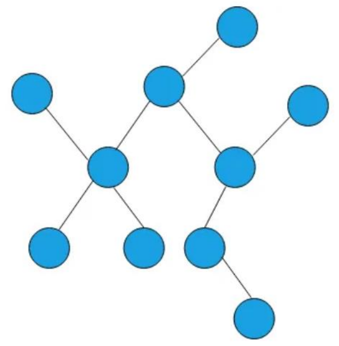


3-coloring rings
Classical: Θ(log*n)
Quantum: ???
Non-signaling: Θ(1)

already obtained in
[Holroyd, Liggett 2016]
[Holroyd, Hutchcroft, Levy 2018]

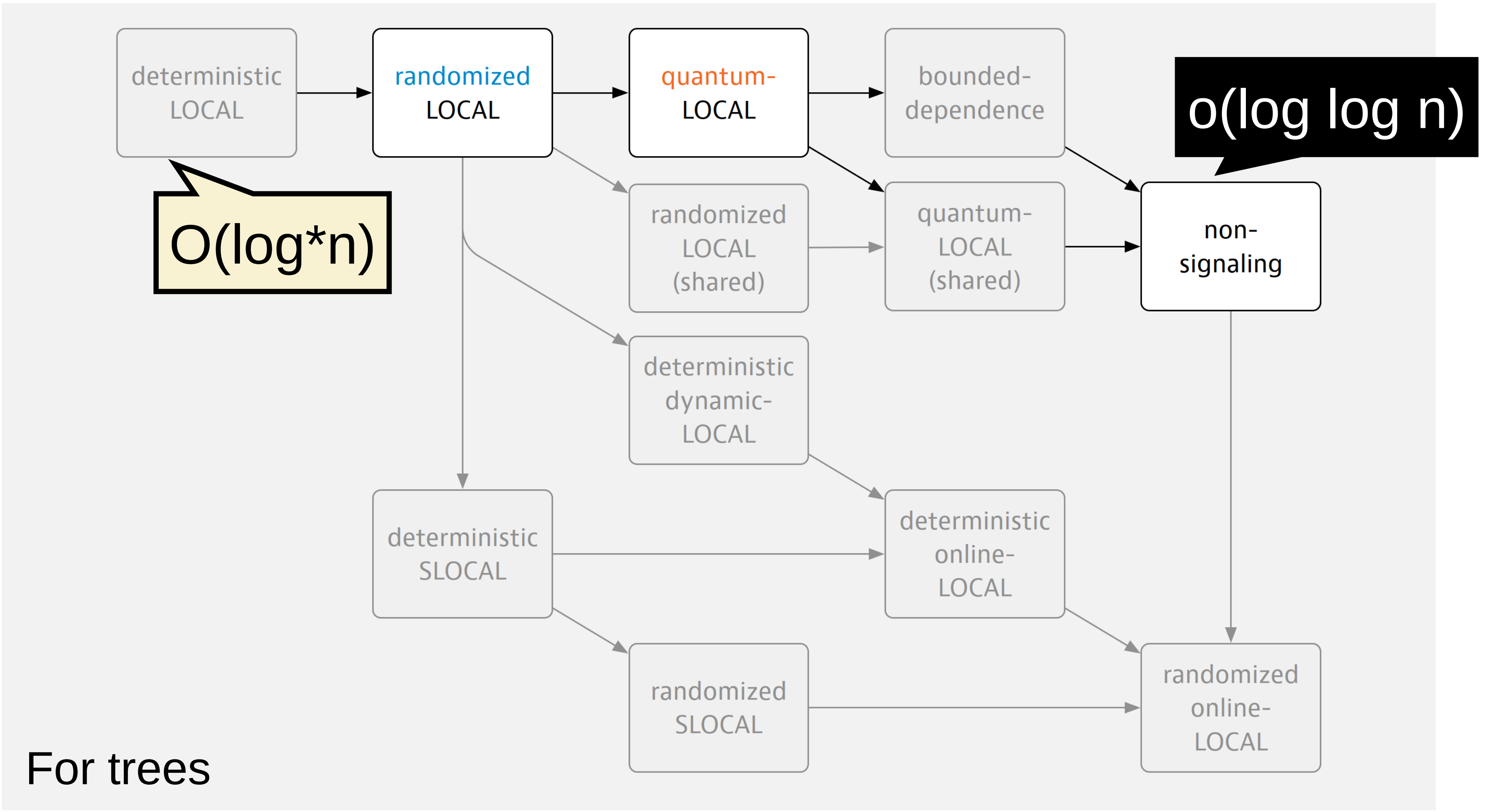
➡ The natural approach to solve the conjecture doesn't work for a very large class of problems (positive interpretation: there might be a quantum advantage!)

Second Result

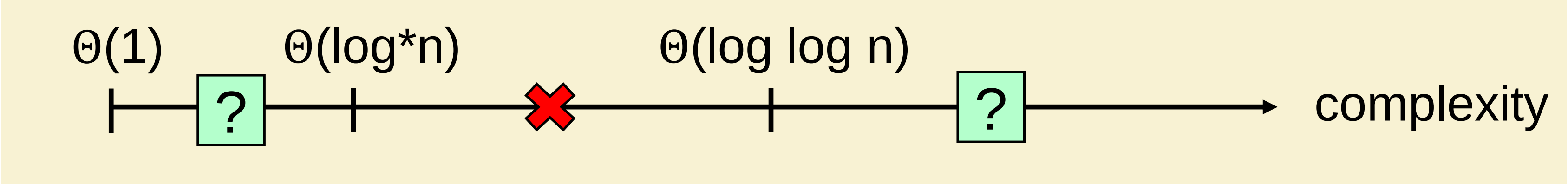


Theorem 2

In trees, any locally-checkable problem that can be solved in o(log log n) rounds in the non-signaling model can be solved in O(log*n) rounds in the deterministic LOCAL model.



⇨ In trees, there is no locally-checkable problem with locality between ω(log*n) and o(log log n) in the quantum LOCAL model



Overview of the Proof of the Main Result

Theorem 1

Any locally-checkable problem that can be solved in o(log n) rounds in the deterministic LOCAL model can be solved in **O(1) rounds in the non-signaling model**.

Tool #1: An **O(1)-round non-signaling strategy** for 3-coloring a ring from [Holroyd, Liggett 2016] [Holroyd, Hutchcroft, Levy 2018]

Tool #2: A reduction [adapted from prior works] from **any locally-checkable problem that can be solved in o(log n) rounds in the deterministic LOCAL model** to the problem of computing a (d+1)-coloring of the graph, where d is the maximum degree of the graph

1. We show that the O(1)-round non-signaling for 3-coloring a ring (Tool #1) can be extended to 3-coloring a "pseudoforest"
2. We observe that all bounded-degree graphs have a "nice" decomposition in pseudoforests
3. We show how to combine 1 and 2 to obtain a (d+1)-coloring for any graph of max degree d
4. The conclusion follows using the reduction from Tool #2

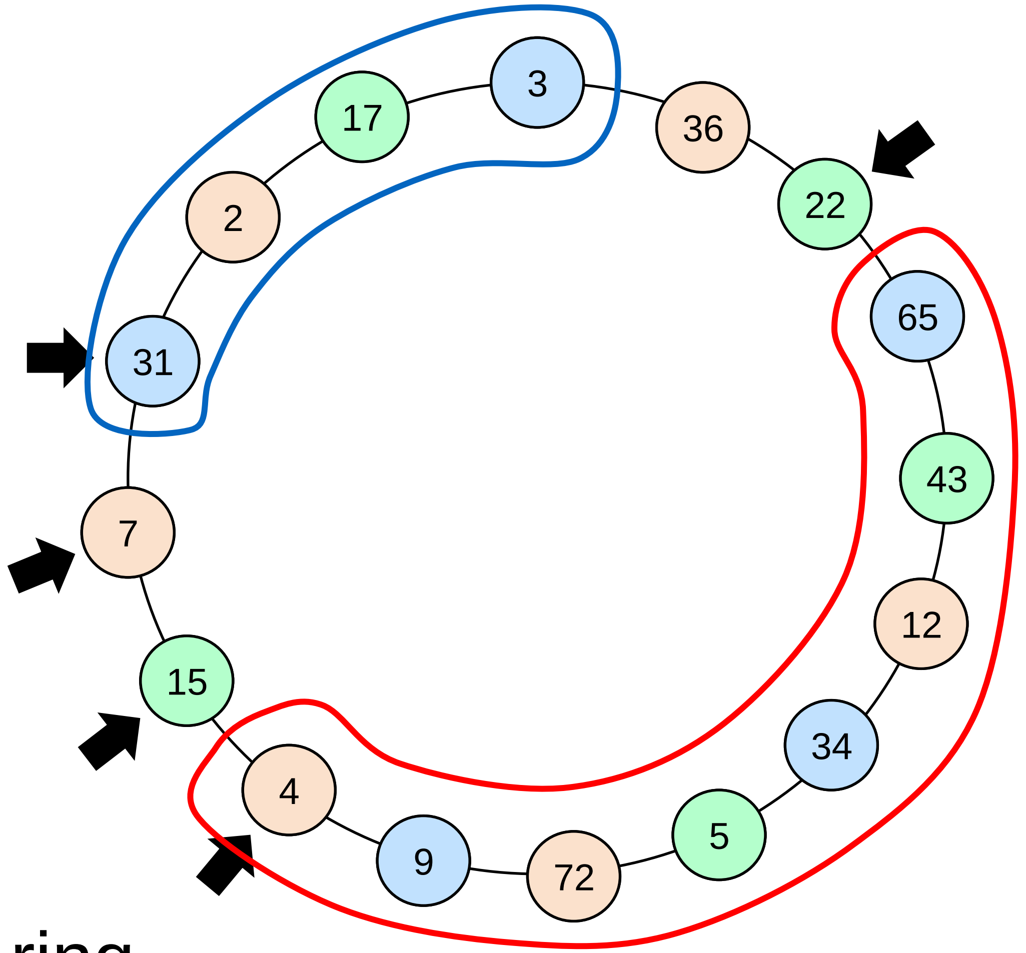
3-Coloring a Ring: Non-Signaling Strategy

[Holroyd, Liggett 2016] [Holroyd, Hutchcroft, Levy 2018]

Consider the following probabilistic process:

1. put a uniformly random color (○,○ or ○) at a uniformly random position
2. put a different uniformly random color at a different uniformly random position
3. repeat n-2 times

   pick a uniformly random node between consecutive colored nodes and insert a color differing from the colors of the two colored neighbors.



This process always produces a valid 3-coloring of the ring

➡ we get a probability distribution over valid 3-colorings of the ring

Theorem ([Holroyd, Liggett 2016] [Holroyd, Hutchcroft, Levy 2018]):

> The restrictions to any two sets of vertices at graph distance greater than 2 are independent of each other.

➡ 1-hop non-signaling distribution

Conclusion

✓ We have shown several relations between the classical, quantum, non-signaling LOCAL models (and many more models)

✓ Main message (for the quantum community):

Theorem 1

Any locally-checkable problem that can be solved in o(log n) rounds in the classical LOCAL model can be solved in O(1) rounds in the non-signaling model.

“For a large class of problems, it is not possible to exclude quantum advantage by using non-signaling arguments”

(positive interpretation: there might be a quantum advantage!)

✓ Open problems

- Prove a quantum advantage for some locally-checkable problem
- Can we exclude quantum advantage for some concrete locally-checkable problem that has classical complexity $\Theta(\log^* n)$? For instance, 3-coloring in rings?
- Does shared entanglement help for any locally-checkable problem?