TWIST FIELD DEFORMATIONS IN STRING FIELD THEORY

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Based on
L. Mattiello and I. Sachs, JHEP11(2019)118

XHZ and I. Sachs, to appear

WHAT IS STRING FIELD THEORY, AND WHAT IS IT GOOD FOR?

- String theory amplitudes are obtained by summing up integrals on moduli spaces of Riemann surfaces
- It is by definition **perturbative** and **background dependent**
- Q: Is it possible to construct an action, from which the string amplitudes can be derived via Feynman diagram expasion?
- Such a theory is the second quantized theory of string theory, which is called **string field theory (SFT)**

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- During the last few decades, we have already developed complete formulations for all the conventional string field theories ($\mathcal{N} = 0$ and $\mathcal{N} = 1$; open, closed and heterotic)
- What were achieved:
  1. Describing **tachyon condensation** (False vacuum)
  2. Essentially field theoretic problems in perturbative thoery: mass renormalization, D-instanton sector contribution, …
  3. Carrying out CFT perturbation theory beyond leading order (closely related to 1.)
- What is not done yet:

  A fully non-perturbative formulation of string theory (By far still a perturbative Batalin-Vilkovisky (BV) action around a saddle point!)

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THE PHYSICS OF CONDENSING TACHYON

- At open bosonic string vacuum (a spacetime-filling D25-brane), a string state

$$\Psi = \int \mathrm{d}k \left\{ \frac{1}{\sqrt{\alpha'}} \left[T(k) + A_\mu(k) a^\mu_{-1} \right] c_0 + \frac{i}{\sqrt{2}} B(k) c_0 + \cdots \right\} |0; k\rangle$$

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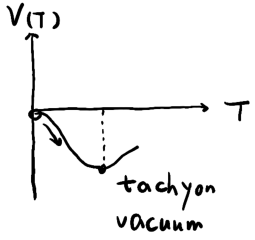
- This means that we are doing perturbation on a wrong vacuum!



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- $T(k)$: tachyon field with **negative** squared mass
- **Sen's conjecture**: the open string vacuum (D25-brane) decays into a vinicity of D-branes, where there are no open strings (**tachyon vacuum**)
  - It was proven by constructing an analytic expression for this "tachyon vacuum" (Schnabl, 05)

THE PHYSICS OF CONDENSING TACHYON

- Another kind of false vacuum: marginal deformation
- **Background independence**: String backgrounds ↔ Classical solutions to SFT ↔ Worldsheet CFT
- A deformation of worldsheet CFT can be represented by an operator of conformal dimension one and it is called an **marginal operator**
- Not every marginal operator represents a valid deformation, those that do are called **exactly marginal**
- Exactly marginal operators are valid string backgrounds, therefore they are solutions to the classical SFT equation of motion

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SFT ACTION

- Open

$$S[\Psi] = \frac{(-1)^{\deg(\Psi)}}{2}\langle\Psi, Q\Psi\rangle + \sum_{n=2}^{\infty}\frac{(-1)^{\deg(\Psi)}}{n+1}\langle\Psi, M_n(\Psi,\ldots,\Psi)\rangle$$

  $\deg(\Psi) = \mathrm{gr}(\Psi) + 1, \Psi \in \mathcal{H}, M_n : \mathcal{H}^{\otimes n} \to \mathcal{H}$ are A_∞ products

- Closed

$$S[\Psi] = \frac{(-1)^{\deg(\Psi)}}{2}\langle\Psi, c_0^- Q\Psi\rangle + \sum_{n=2}^{\infty}\frac{(-1)^{\deg(\Psi)}}{(n+1)!}\langle\Psi, c_0^- L_n(\Psi,\ldots,\Psi)\rangle$$

  $\deg(\Psi) = \mathrm{gr}(\Psi), \Psi \in \{\Psi \in \mathcal{H} \mid b_0^-\Psi = L_0^-\Psi = 0\}, L_n : \mathcal{H}^{\wedge n} \to \mathcal{H}$ are L_∞ products

TWIST FIELD IN OPEN THEORY: ADHM INSTANTON

- k D(-1)-branes sitting on a stack of N D3-branes

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- Field theoretically, this is a $\mathcal{N} = 4, SU(N)$ super Yang-Mills instanton with winding number k
- But this is not exactly true: D-brane bound system is an approximated string background, just like point-like instanton is an approximated solution

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- The operator relevant here:

$$\Delta(z) = \sigma^0(z)\sigma^1(z)\sigma^2(z)\sigma^3(z),$$

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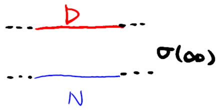
- $\sigma^{\mu}(z)$ is the twist field that change the boundary condition of strings streching between different types of D-branes



TWIST FIELD IN OPEN THEORY: ADHM INSTANTON

- The twist fields can be made into an marginal operator

$$V(z) = \frac{g_{YM}}{\sqrt{\alpha'}} c(z) \begin{pmatrix} A^{uv}_{\mu}\psi^{\mu} & w^{uj}_{\dot{\alpha}}\Delta S^{\dot{\alpha}} \\ \bar{w}^{iv}_{\dot{\alpha}} S\bar{\Delta} & a^{ij}_{\mu}\psi^{\mu} \end{pmatrix} (z) e^{-\phi}(z),$$

  where $u, v \in \{1, \ldots, N\}$ and $i, j \in \{1, \ldots, k\}$.
- It satisfies $QV = 0$
- The full string field equation of motion

$$Q\Psi + M_2(\Psi, \Psi) + M_3(\Psi, \Psi, \Psi) + \cdots = 0$$

  is satisfied only if we deform V

$$\Psi = V + \left(\frac{\rho}{\sqrt{\alpha'}}\right)^2 \Psi_1 + \left(\frac{\rho}{\sqrt{\alpha'}}\right)^3 \Psi_2 + \cdots$$

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TWIST FIELD IN OPEN THEORY: ADHM INSTANTON

- At each order

$$Q\Psi_0 = 0, V \equiv \frac{\rho}{\sqrt{\alpha'}}\Psi_0$$

$$Q\Psi_1 + M_2(\Psi_0, \Psi_0) = 0$$

$$Q\Psi_2 + M_2(\Psi_0, \Psi_1) + M_2(\Psi_1, \Psi_0) + M_3(\Psi_0, \Psi_0, \Psi_0) = 0$$

$$\dots$$

Warning: Q is not invertible

- Sovable condition (P_0 is a projector):

$$P_0 M_2(\Psi_0, \Psi_0) = 0$$

$$P_0 \left[M_2(\Psi_1, \Psi_0) + M_2(\Psi_0, \Psi_1) + M_3(\Psi_0 + \Psi_0 + \Psi_0)\right] = 0$$

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- For $SU(2)$ group

$$A_\mu^{c(1)} = \left(\frac{\rho}{\sqrt{\alpha'}}\right)^2 (\mathcal{V}_{A_\mu^c}, \Psi_1) = 2\rho^2 \bar{\eta}^c_{\mu\nu} \frac{(x-x_0)^\nu}{(x-x_0)^4},$$

which agrees with the ADHM construction (expanded for small ρ)

- The sovable condition gives

$$\bar{\eta}_a^{\mu\nu}\left(\left[a_\mu, a_\nu + \frac{1}{2}\bar{w}_{\dot\alpha}(\bar{\sigma}_{\mu\nu})^{\dot\alpha\dot\beta} w_{\dot\beta}\right]\right) = 0,$$

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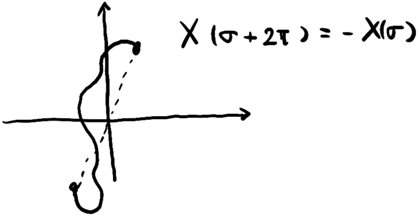
TWIST FIELD IN CLOSED THEORY: ORBIFOLD SINGULARITY

- Deformation is given by $\tilde{c}cw_\beta S^\beta \Delta \tilde{w}_\beta \tilde{S}^\beta \tilde{\Delta} e^{-\phi} e^{-\tilde{\phi}}$



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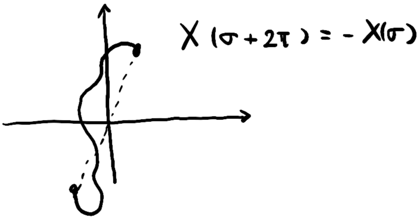

- There is no constraint on the moduli (up to the third order)
- So we can take $w_\alpha = \bar{w}_\alpha = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

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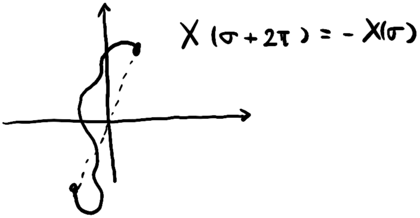

- Metric is given by

$$G^{(1)}_{I\bar{J}} = \left(\frac{\rho^2}{\alpha'}\right)^2 (\mathcal{V}_{G_{I\bar{J}}}, \Psi_1) \sim \begin{pmatrix} 2ik_{\bar{1}}k_{\bar{1}} & 2k_{\bar{1}}k_1 & -2ik_{\bar{1}}k_{\bar{0}} & -2k_{\bar{1}}k_0 \\ 2k_{\bar{1}}k_1 & -2ik_1k_1 & -2k_{\bar{0}}k_1 & 2ik_0k_1 \\ -2ik_{\bar{1}}k_{\bar{0}} & -2k_{\bar{0}}k_1 & 2ik_{\bar{0}}k_{\bar{0}} & 2k_{\bar{0}}k_0 \\ -2k_{\bar{1}}k_0 & 2ik_0k_1 & 2k_{\bar{0}}k_0 & -2ik_0k_0 \end{pmatrix}$$

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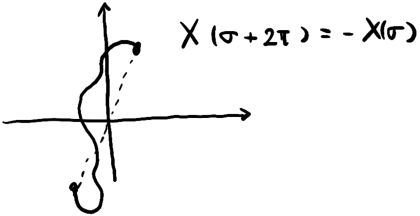

- Kähler form: $\omega = iG_{I\bar{J}} \mathrm{d}x^I \wedge \mathrm{d}x^{\bar{J}}$, computing $\mathrm{d}\omega$ will give a non-vanishing result

TWIST FIELD IN CLOSED THEORY: ORBIFOLD SINGULARITY

- Relation to the resolved space?
- Will there be contraints at higher order?

Thank you!