The State-refocusing Square Root Instrument and Retrodictive Entropic Uncertainty Relations

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Abstract

[This work is in progress now.] Quantum measurement, as an interesting topic that shows one of the main differences between classical and quantum physics, is widely studied. One of the most famous quantum relations that are studied in quantum measurement is the uncertainty relations, which was first given by Heisenberg [1]. The Heisenberg's uncertainty relations states that, it is impossible to correctly measure the position and the momentum of a particle simultaneously. More formally, Heisenberg's uncertainty principle can be expressed by: $\varepsilon(Q)\eta(P) \geq \frac{\hbar}{2}$ and is latter be extended by Robertson [2] as: $\sigma(Q)\sigma(P) \geq \frac{1}{2}|\langle\psi|[A,B]|\psi\rangle|$, where $\sigma(Q)$ and $\sigma(P)$ are standard deviation and $[A,B] = \mathrm{AB} - \mathrm{BA}$ is the commutator. With the uncertainty of the measurement outcome and the disturbance of measurement, it is interesting to investigate the interplay of quantum measurement and so as the quantum correlations.

When considering the general measurements, we usually express them by Positive Operator-Valued Measure (POVM) $M = \{M_i\}_{i\in\mathcal{I}}$ and the quantum instrument $\mathcal{J}$ that correlated with measurement M. In this talk, we consider a special type of quantum instrument that we name as "state-refocus square root instrument" which is inspired by Fuchs [3]. It is a special form of square root instrument that gives the same joint probability distribution and the marginal probability distribution of two sequential measurements $\{M_x\}$ and $\{N_y\}$ (generally can be the same POVM) no matter what measuring order we choose. We found the SRSR-Instrument very interesting to be considered because this kind of relation implies the minimum disturbance of a measurement. Inspired by Appleby [4] and Ge Bai [5], we start to consider the "Retrodictive Entropic Uncertainty Relations" by using SRSR-Instrument, because our scenario can be considered as applying a recovery map whose prior is the original state ρ after the first measurement, which is considered as the Quantum Bayes' rule in [5] and furthermore is a "retrodiction" of the state before the measurement [4]. We consider entropic uncertainty relations instead of the standard deviation uncertainty relations because it avoid many practical interpretation issues and is natural from quantum information theory [6]. We applied our scenario to the entropic uncertainty relations and did some numerical experiments. It shows that our scenario can get a tighter lower bound of entropic uncertainty relations than the previous one that was given by Berta [7] in most of the cases, except when the number of POVM's outcomes is small (2 outcomes). We are working on figuring out a new entropic uncertainty relations using the idea of "retrodictive error". We believe our SRSR-Instrument will give a tight bound because this kind of quantum instrument gives the minimum disturbance on quantum state.

Reference

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