ADHM Construction of Instantons (Review)

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1. Introduction

- Instantons are global solutions of Anti-Self-Dual (ASD) Yang-Mills eqs. with finite action.
- Instantons play crucial roles in quantum field theory (QFT) and geometry.
- ADHM(=Atiyah-Drinfeld-Hitchin-Manin) construction is a powerful method to construct the instanton solutions which is based on one-to-one correspondence between moduli spaces of the instantons and ADHM data.

ASDYM eq. in 4-dim. with G=U(N)

- **ASDYM eq. (real rep.)**

$\mu, \nu = 1,2,3,4$

$$F_{12} = -F_{34}, \quad F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu + A_\mu A_\nu - A_\nu A_\mu$$

Field strength

$$F_{13} = -F_{42},$$

$$F_{14} = -F_{23}.$$

A_μ: **Gauge field (N × N anti-Hermitian)**

$$(\Leftrightarrow F_{z_1\bar{z}_1} + F_{z_2\bar{z}_2} = 0, \quad F_{z_1 z_2} = 0 \quad (cpx.\, rep.))$$

- **Instanton number (2nd Chern number):**

$$C_2[A_\mu] = \frac{1}{8\pi^2}\int \mathrm{Tr} F \wedge F \quad \in \mathbb{Z}$$

2. Atiyah-Drinfeld-Hitchin-Manin Construction

based on duality for the instanton moduli space

4dim. ASDYang-Mills eq. (Difficult)

$$F_{z_1\bar{z}_1} + F_{z_2\bar{z}_2} = 0$$
$$F_{z_1 z_2} = 0$$

N × N PDE

ADHM eq. (≒0dim. ASDYM) (Easy)

$$[B_1, B_1^+] + [B_2, B_2^+] + I I^+ - J^+ J = 0$$
$$[B_1, B_2] + IJ = 0$$

k × k Matrix eqs.

1:1

Sol.= instantons (G=U(N), C_2 =k) ⟷ **Sol.=ADHM data (G=`U(k)')**

$$A_\mu : N \times N$$

$$B_{1,2} : k \times k, \quad I : k \times N, \quad J : N \times k$$

Gauge trf.:

$$A_\mu \mapsto g^{-1} A_\mu g + g^{-1} \partial_\mu g$$
$$g \in U(N)$$

Gauge trf.:

$$B_{1,2} \mapsto \tilde{g}^{-1} B_{1,2} \tilde{g}, \quad \tilde{g} \in U(k)$$
$$I \mapsto \tilde{g}^{-1} I, \quad J \mapsto J\tilde{g}$$

Fourier-Mukai-Nahm transformation

Beautiful duality between instanton moduli on 4-tori and instanton moduli on the dual tori

4dim. ASDYang-Mills eq. on a 4-torus

$$F_{z_1\bar{z}_1} + F_{z_2\bar{z}_2} = 0$$
$$F_{z_1 z_2} = 0$$

N × N PDE

G (→)

F (←)

4dim. ASD Yang-Mills eq. on the dual torus

$$\widetilde{F}_{\xi_1\bar{\xi}_1} + \widetilde{F}_{\xi_2\bar{\xi}_2} = 0$$
$$\widetilde{F}_{\xi_1\xi_2} = 0$$

k × k PDE

Sol.=instantons (G=U(N), C_2 =k)

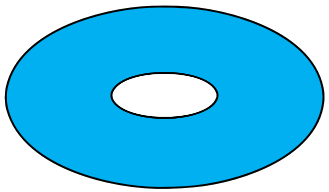
1:1

Sol.=the dual instantons (G=U(k), C_2=N)

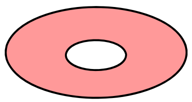
$A_\mu : N \times N$

⇕

$\widetilde{A}_\mu : k \times k$



Define the maps F & G, & G∘F=id. & F∘G=id.



On a 4-torus

On the dual 4-torus

Fourier-Mukai-Nahm transformation

Beautiful duality between instanton moduli on 4-tori and instanton moduli on the dual tori

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N × N PDE

4dim. ASD Yang-Mills eq. on the dual torus

$$\widetilde{F}_{\xi_1\bar{\xi}_1} + \widetilde{F}_{\xi_2\bar{\xi}_2} = 0$$

$$\widetilde{F}_{\xi_1\xi_2} = 0$$

k × k PDE

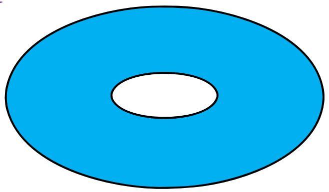
1:1

Sol.=instantons (G=U(N), C_2 =k) ⟷ Sol.=the dual instantons (G=U(k), C_2=N)

map F (Dirac eq.)

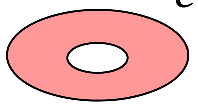
$$A_\mu(x) = \left\langle V, \partial_\mu V \right\rangle_\xi \quad \longleftarrow \quad \widetilde{A}_\mu(\xi) : k \times k$$

N × N



$$\nabla^+ V = \bar{e}^\mu \otimes \left(\frac{\partial}{\partial \xi^\mu} + \widetilde{A}_\mu - i x_\mu\right) V = 0$$

$$\bar{e}^\mu := (i\sigma_a, 1_2)$$



$$V : 2k \times \underline{N}$$

Family index thm.

On a 4-torus $: x_\mu$

On the dual 4-torus $: \xi_\mu$

Fourier-Mukai-Nahm transformation

Beautiful duality between instanton moduli on 4-tori and instanton moduli on the dual tori

4dim. ASDYang-Mills eq. on a 4-torus

$$F_{z_1\bar{z}_1} + F_{z_2\bar{z}_2} = 0$$
$$F_{z_1 z_2} = 0$$

N × N PDE

1:1

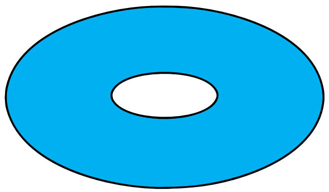
4dim. ASD Yang-Mills eq. on the dual torus

$$\widetilde{F}_{\xi_1\bar{\xi}_1} + \widetilde{F}_{\xi_2\bar{\xi}_2} = 0$$
$$\widetilde{F}_{\xi_1\xi_2} = 0$$

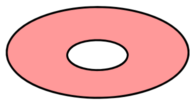
k × k PDE

Sol.=instantons (G=U(N), C_2 =k) ⟷ Sol.=the dual instantons (G=U(k), C_2=N)

$A_\mu(x) : N \times N$ —— map G (Dirac eq.) ——→ $\widetilde{A}_\mu(\xi) = \left\langle \psi , \widetilde{\partial}_\mu \psi \right\rangle_\xi$ k×k



$$\bar{e}_\mu D_\mu \psi = \bar{e}^\mu \otimes \left(\frac{\partial}{\partial x^\mu} + A_\mu - i\xi_\mu\right)\psi = 0$$



$$\psi : 2N \times \underline{k}$$

On a 4-torus $: x_\mu$ Family index thm. **On the dual 4-torus** $: \xi_\mu$

Fourier-Mukai-Nahm transformation

Beautiful reciprocity between instanton moduli on 4-tori and instanton moduli on the dual tori

4dim. ASDYang-Mills eq. on a 4-torus

$$F_{z_1\bar{z}_1} + F_{z_2\bar{z}_2} = 0$$

$$F_{z_1 z_2} = 0$$

N × N PDE

Dirac eq. G →

← F Dirac eq.

4dim. ASD Yang-Mills eq. on the dual torus

$$\widetilde{F}_{\xi_1\bar{\xi}_1} + \widetilde{F}_{\xi_2\bar{\xi}_2} = 0$$

$$\widetilde{F}_{\xi_1\xi_2} = 0$$

k × k PDE

Sol.=instantons (G=U(N), C_2 =k)

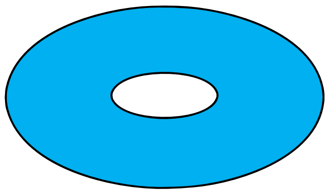
1:1 (reciprocity)

Sol.=the dual instantons (G=U(k), C_2=N)

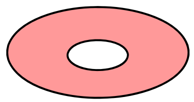
$A_\mu : N \times N$

⇕

$\widetilde{A}_\mu : k \times k$



Define the maps F & G, & G∘F=id. & F∘G=id.



On a 4-torus

On the dual 4-torus

Fourier-Mukai-Nahm trf. (radii of the torus→∞)

reciprocity between instanton moduli on R^4 and instanton moduli on ``1pt.'' [cf. van Baal, hep-th/9512223]

4dim. ASDYang-Mills eq.

$$F_{z_1\bar{z}_1} + F_{z_2\bar{z}_2} = 0$$

$$F_{z_1 z_2} = 0$$

N × N PDE

1:1

0dim. ASD Yang-Mills eq.

$$\widetilde{F}_{\mu\nu} := \cancel{\partial_\mu}\widetilde{A}_\nu - \cancel{\partial_\nu}\widetilde{A}_\mu + [\widetilde{A}_\mu, \widetilde{A}_\nu]$$

$$\widetilde{F}_{\xi_1\bar{\xi}_1} + \widetilde{F}_{\xi_2\bar{\xi}_2} = 0$$

$$\widetilde{F}_{\xi_1\xi_2} = 0$$

Matrix eq. !

k × k ~~PDE~~

Sol.=instantons
(G=U(N), C_2 =k)

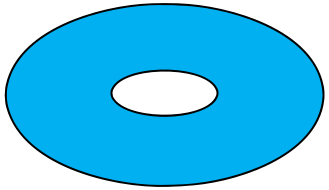
⟷

Sol.=``dual instantons''
(G=U(k), ``C_2=N'')

map F (0dim Dirac eq.) ←

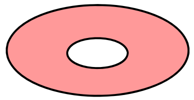
$$A_\mu = V^+ \partial_\mu V$$

$$\widetilde{A}_\mu : k \times k$$



$$\nabla^+ V = \bar{e}^\mu \otimes \left(\cancel{\frac{\partial}{\partial \xi^\mu}} + \widetilde{A}_\mu - i x_\mu\right) V = 0$$

Matrix eq. !



$$V : 2k \times \underline{N}$$

Linear alg.

On a 4-torus→R^4

On the dual 4-torus→1 pt.

Atiyah-Drinfeld-Hitchin-Manin (ADHM) Construction
based on the following reciprocity

4dim. ASDYang-Mills eq. — ADHM eq. (≒0dim. ASDYM)

$$F_{z_1\bar{z}_1} + F_{z_2\bar{z}_2} = 0$$
$$F_{z_1 z_2} = 0$$

N × N PDE

G(4dim D.eq.) →

← F(0dim D.eq.)

$$[B_1, B_1^+] + [B_2, B_2^+] + I I^+ - J^+ J = 0$$
$$[B_1, B_2] + IJ = 0$$

k × k matrix eq.

Sol.=instantons (G=U(N), C_2 =k)

$A_\mu : N \times N$

1:1 ↔

Proved in the same way as the Nahm trf.

Sol.=ADHM data (G=`U(k)’)

$B_{1,2} : k \times k,$

$I : k \times N, \quad J : N \times k$

Other limits and related works

- 3radii→∞&1radius→0 : [Nahm, Hitchin, Nakajima,…]

  monopole on 3-dim ←→ Nahm data on 1-dim
- 2radii→∞&2radii→0 :

  Hitchin system on 2dim ←→ Hitchin system 2dim
- 3radii→∞&finite1radius : [Hurtubise-Murray,…]

  caloron on R^2x S^1 ←→ Nahm data on S^1
- 2radii→∞&finite2radii : : [Jardim, Mochizuki, …]

  instanton on R^2x T^2 ←→ Hitchin system on T^2
- 1radii→∞&finite3radii : : [Charbonneau, Yoshino,…]

  instanton on R x T^3 ←→ monopole on T^3

Atiyah-Drinfeld-Hitchin-Manin (ADHM) Construction

based on the following reciprocity

4dim. ASDYang-Mills eq.

$$F_{z_1\bar{z}_1} + F_{z_2\bar{z}_2} = 0$$

$$F_{z_1 z_2} = 0$$

N × N PDE

G(4dim D.eq.) →

← F(0dim D.eq.)

ADHM eq.. (≒0dim. ASDYM)

RHS is in fact $[z_1, \bar{z}_1] + [z_2, \bar{z}_2]$

$$[B_1, B_1^+] + [B_2, B_2^+] + I\,I^+ - J^+ J = 0$$

$$[B_1, B_2] + IJ = 0$$

k × k matrix eq.

Sol.=instantons
(G=U(N), C_2 =k)

$$A_\mu : N \times N$$

1:1

Proved in the same way as the Nahm trf.

Sol.=ADHM data
(G=`U(k)’)

$$B_{1,2} : k \times k,$$

$$I : k \times N, \quad J : N \times k$$

ADHM(Atiyah-Drinfeld-Hitchin-Manin) construction

Ex.) Commutative BPST instanton（N=2, k=1）

4dim. ASDYang-Mills eq.

$$F_{z_1\bar{z}_1} + F_{z_2\bar{z}_2} = 0$$
$$F_{z_1 z_2} = 0$$

N × N PDE

ADHM eq. (≒0dim. ASDYM)

$$[B_1, B_1^+] + [B_2, B_2^+] + I\,I^+ - J^+ J = 0$$
$$[B_1, B_2] + IJ = 0$$

k × k matrix eq.

1:1

BPST instanton
(G=U(2), C_2 =1)

Sol.=ADHM data
(G=`U(1)’)

$$A_\mu = \frac{i(x-b)^\nu \eta_{\mu\nu}^{ASD}}{(z-\alpha)^2 + \rho^2}, \quad 2\times 2$$

$$F_{\mu\nu} = \frac{2i\rho^2}{((z-\alpha)^2+\rho^2)^2}\eta_{\mu\nu}^{ASD}$$

$$B_{1,2} = \alpha_{1,2}, \quad 1\times 1$$

$$I = (\rho, 0),\ J = \begin{pmatrix} 0 \\ \rho \end{pmatrix}$$

ADHM(Atiyah-Drinfeld-Hitchin-Manin) construction

Ex.) Commutative BPST instanton（N=2, k=1）

4dim. ASDYang-Mills eq.

$$F_{z_1\bar{z}_1} + F_{z_2\bar{z}_2} = 0$$

$$F_{z_1 z_2} = 0$$

N × N PDE

ADHM eq. (≒0dim. ASDYM)

$$[B_1, B_1^+] + [B_2, B_2^+] + I I^+ - J^+ J = 0$$

$$[B_1, B_2] + IJ = 0$$

k × k matrix eq.

1:1

BPST instanton
(G=U(2), C_2 =1)

$$A_\mu = \frac{i(x-b)^\nu \eta_{\mu\nu}^{ASD}}{(z-\alpha)^2 + \rho^2}, \quad 2\times 2$$

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Sol.=ADHM data
(G=`U(1)’)

$$B_{1,2} = \alpha_{1,2}, \quad 1\times 1$$

$$I = (\rho, 0),\ J = \begin{pmatrix} 0 \\ \rho \end{pmatrix}$$

``The first time abstract modern mathematics had been of any use!” (Polyakov)

ADHM(Atiyah-Drinfeld-Hitchin-Manin) construction

Ex.) Commutative BPST instanton（N=2, k=1）

4dim. ASDYang-Mills eq.

$$F_{z_1\bar{z}_1} + F_{z_2\bar{z}_2} = 0$$
$$F_{z_1 z_2} = 0$$

N × N PDE

ADHM eq. (≒0dim. ASDYM)

$$[B_1, B_1^+] + [B_2, B_2^+] + I I^+ - J^+ J = 0$$
$$[B_1, B_2] + IJ = 0$$

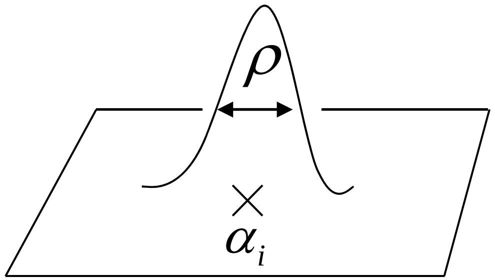
k × k matrix eq.

1:1

BPST instanton (G=U(2), C_2 =1) ⟷ **Sol.=ADHM data (G=\`U(1)’)**

$$A_\mu = \frac{i(x-b)^\nu \eta_{\mu\nu}^{ASD}}{(z-\alpha)^2 + \rho^2}, \quad 2\times 2$$

$$F_{\mu\nu} = \frac{2i\rho^2}{((z-\alpha)^2+\rho^2)^2}\eta_{\mu\nu}^{ASD}$$



position

$$B_{1,2} = \alpha_{1,2}, \quad 1\times 1$$

$$I = (\rho, 0),\ J = \begin{pmatrix} 0 \\ \rho \end{pmatrix}$$

size

ADHM(Atiyah-Drinfeld-Hitchin-Manin) construction

Ex.) Commutative BPST instanton（N=2, k=1）

4dim. ASDYang-Mills eq.

$$F_{z_1\bar{z}_1} + F_{z_2\bar{z}_2} = 0$$

$$F_{z_1 z_2} = 0$$

N × N PDE

ADHM eq. (≒0dim. ASDYM)

$$[B_1, B_1^+] + [B_2, B_2^+] + I I^+ - J^+ J = 0$$

$$[B_1, B_2] + IJ = 0$$

k × k matrix eq.

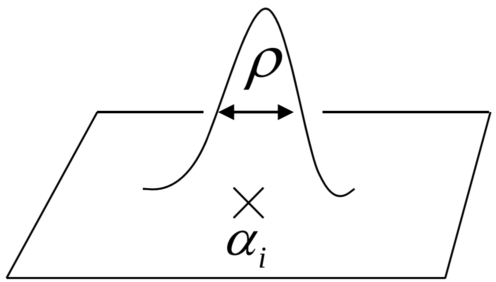
1:1

↔

BPST instanton
(G=U(2), C_2 =1)

$$A_\mu = \frac{i(x-b)^\nu \eta_{\mu\nu}^{ASD}}{(z-\alpha)^2 + \rho^2}, \quad 2 \times 2$$

$$F_{\mu\nu} = \frac{2i\rho^2}{((z-\alpha)^2 + \rho^2)^2} \eta_{\mu\nu}^{ASD}$$



$\rho \to 0$: singular

Sol.=ADHM data
(G=`U(1)’)

position

$$B_{1,2} = \alpha_{1,2}, \quad 1 \times 1$$

$$I = (\rho, 0),\ J = \begin{pmatrix} 0 \\ \rho \end{pmatrix}$$

size

ADHM(Atiyah-Drinfeld-Hitchin-Manin) construction

Ex.) Commutative BPST instanton（N=2, k=1）

4dim. ASDYang-Mills eq.

$$F_{z_1\bar{z}_1} + F_{z_2\bar{z}_2} = 0$$

$$F_{z_1 z_2} = 0$$

N × N PDE

BPST instanton (G=U(2), C_2 =1)

$$A_\mu = \frac{i(x-b)^\nu \eta^{ASD}_{\mu\nu}}{(z-\alpha)^2+\rho^2}, \quad 2\times 2$$

$$F_{\mu\nu} = \frac{2i\rho^2}{((z-\alpha)^2+\rho^2)^2}\eta^{ASD}_{\mu\nu}$$

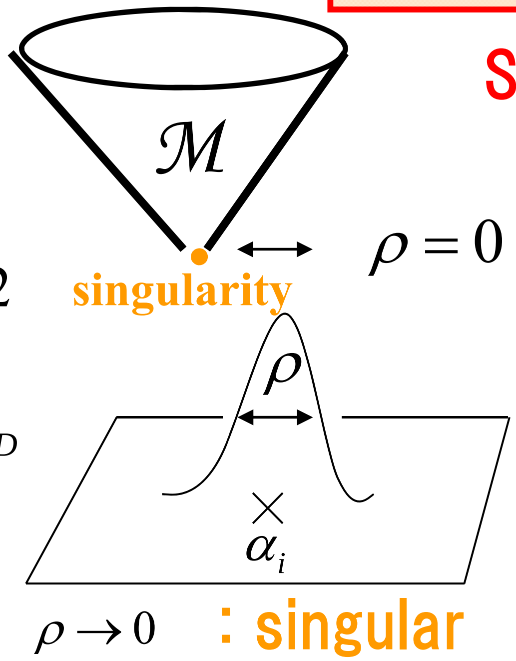
ADHM eq. (≒0dim. ASDYM)

$$\mu_R = [B_1, B_1^+] + [B_2, B_2^+] + I I^+ - J^+ J = 0$$

$$\mu_C = [B_1, B_2] + IJ = 0$$

k × k matrix eq.

Sol.=ADHM data (G=`U(1)’)



position

$$B_{1,2} = \alpha_{1,2}, \quad 1\times 1$$

$$I = (\rho, 0),\ J = \begin{pmatrix} 0 \\ \rho \end{pmatrix}$$

size

ADHM(Atiyah-Drinfeld-Hitchin-Manin) construction

Ex.) NC BPST instanton（N=2, k=1）

NC ASDYang-Mills eq.

$$F_{z_1\bar{z}_1} + F_{z_2\bar{z}_2} = 0$$

$$F_{z_1 z_2} = 0$$

N × N PDE

NC ADHM eq.

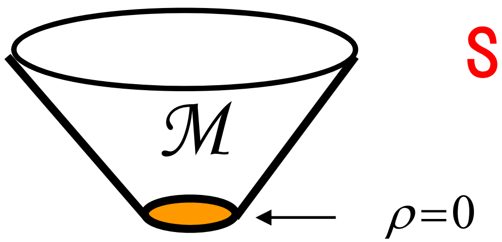
$$\mu_R = [B_1, B_1^+] + [B_2, B_2^+] + I I^+ - J^+ J = \zeta$$

$$\mu_C = [B_1, B_2] + IJ = 0$$

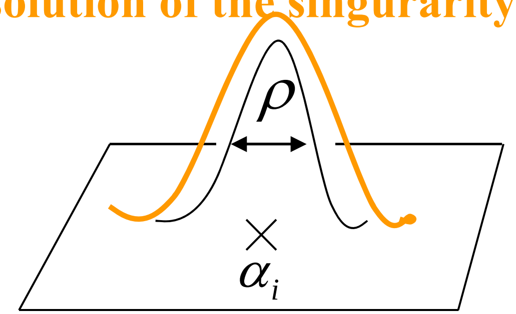
k × k marix eq.

NC BPST instanton
(G=U(2), C_2 =1)

$A_\mu, F_{\mu\nu}$: exact sol.



Resolution of the singurarity!



$\rho \to 0$: regular!

Sol.：ADHM data
(G=`U(1)')

position

$$B_{1,2} = \alpha_{1,2},$$

$$I = (\sqrt{\rho^2 + \zeta}, 0),\ J = \begin{pmatrix} 0 \\ \rho \end{pmatrix}$$

size　Fat by ζ !

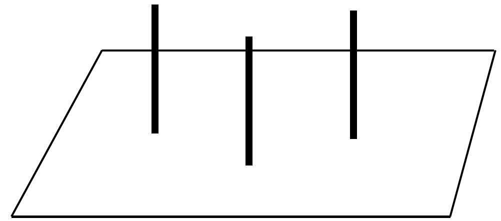
The instanton moduli space

- **in commutative spaces**

$$\overline{\mathrm{M}}_{k,N} = \mathrm{M}_{k,N} \cup (\mathrm{M}_{k-1,N} \times R^4) \cup (\mathrm{M}_{k-2,N} \times Sym^2 R^4) \cup \ldots$$
$$\cup (\mathrm{M}_{1,N} \times Sym^{k-1} R^4) \cup \underline{Sym^k R^4}$$

k size-zero instantons

Symmetric Product



- **In noncommutative spaces**

$$\overline{\mathrm{M}}_{k,N} = \mathrm{M}_{k,N} \cup (\mathrm{M}_{k-1,N} \times R^4) \cup (\mathrm{M}_{k-2,N} \times \widetilde{Sym^2 R^4}) \cup \ldots$$
$$\cup (\mathrm{M}_{1,N} \times \widetilde{Sym^{k-1} R^4}) \cup \underline{\widetilde{Sym^k R^4}}$$

k U(1) instantons

Hilbert Scheme

- **dim** $\mathrm{M}_{k,N} = 2 \cdot 2k^2 + 2 \cdot 2Nk - 3k^2 - k^2 = 4Nk$ 算数

§ 3 Nahm Construction of Monopoles (G=U(2))

Duality between Bogomol'nyi eq. and Nahm eq. (cf. ADHM Construction of instanton)

Bogomol'nyi eq.(3d ASD)

非線形偏微分方程式(難)

$$B_i = D_i \Phi, \qquad x \in R^3$$

$$\Phi \approx \left(a - \frac{k}{2r} \right) \sigma_3 + O(r^{-2})$$

1:1 ⟷

Nahm eq.(≒1次元ASDYM)

常微分方程式(易)

$$\frac{dT_i}{d\xi} = i\varepsilon_{ijk} T_j T_k, \qquad \xi \in [-a, a]$$

$$T_i \approx \frac{\tau_i}{\xi \mp a} + (\ reg..), \quad [\tau_i, \tau_j] = i\varepsilon_{ijk} \tau_k$$

解:Monopole

G=SU(2) $(\Phi, A_i) : 2 \times 2$

Gauge trf.:

$$A_i \mapsto g^{-1} A_i g + g^{-1} \partial_i g,$$

$$\Phi \mapsto g^{-1} \Phi g, \quad g \in U(2)$$

解:Nahm data

G=U(k) $T_i : k \times k$

Gauge trf.:

$$T_i \mapsto \tilde{g}^{-1} T_i \tilde{g}, \quad \tilde{g} \in U(k)$$

Nahm construction of (NC) monopoles

Nahm, in ``Monopoles in QFT''(1982)

D-brane's interpretation

Diaconescu

Nahm eq. (G=``U(k)''): k × k ODE

$$\frac{dT_i}{d\xi} = i\varepsilon_{ijk}[T_j, T_k]$$

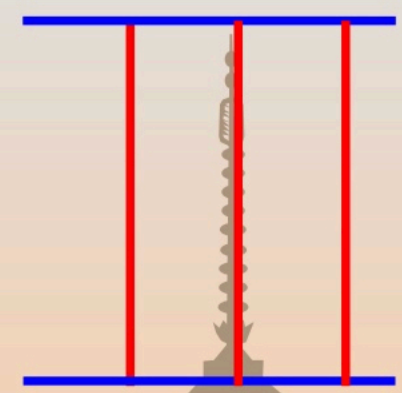
BPS

Nahm data $T_i : k \times k$

1:1

monopoles $\Phi,\ A_i : N \times N$

k D1-branes



N D3-branes

Bogomol'nyi eq. (G=U(N), k): N × N PDE

$$B_i = \partial_i \Phi$$

BPS

A T-dualized configration

Nahm construction of Dirac monopoles

Nahm eq. (G=``U(1)''): 1 × 1 ODE

$$\frac{dT_i}{d\xi} = i\varepsilon_{ijk}[T_j, T_k]$$

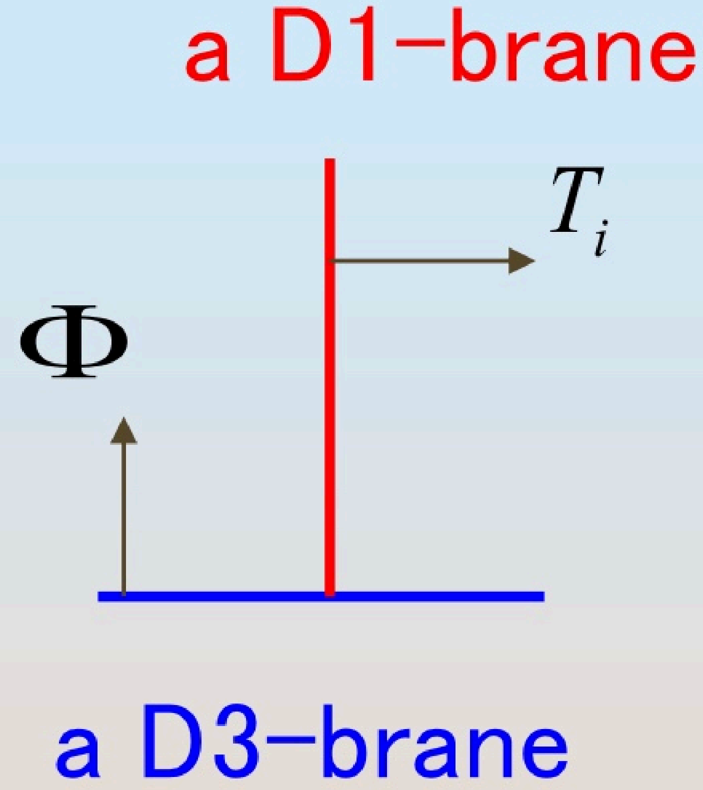
Nahm data $T_i = 0$

1:1

monopoles

Bogomol'nyi eq. (G=U(1), k=1): 1 × 1 PDE

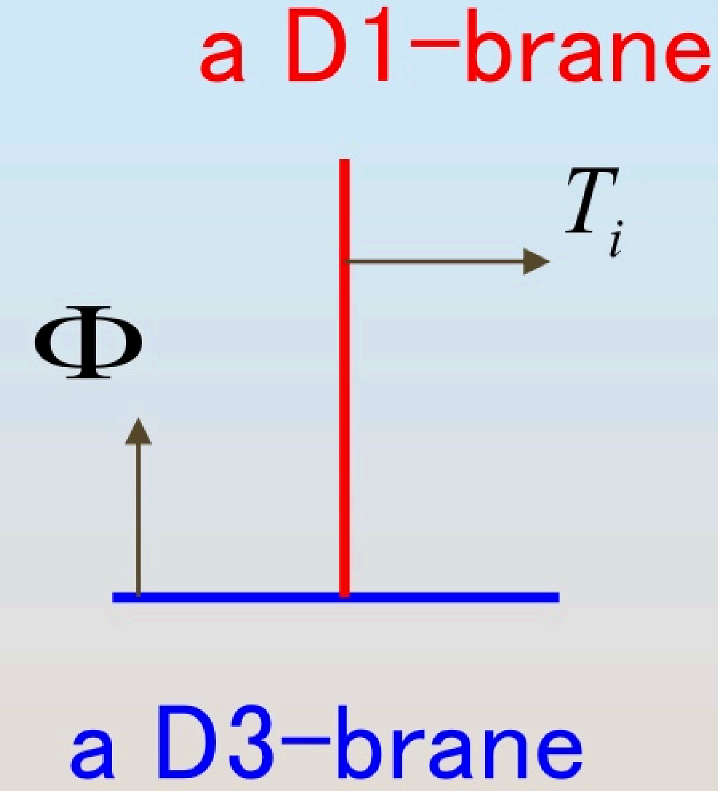
$$B_i = \partial_i \Phi$$



Nahm construction of Dirac monopoles

Nahm eq. (G=``U(1)''): 1 × 1 ODE

$$\frac{dT_i}{d\xi} = i\varepsilon_{ijk}[T_j, T_k]$$



Nahm data $T_i = 0$

1:1

monopoles $\Phi = \frac{1}{2r}, \; A_r = A_\vartheta = 0, \; A_\varphi = \frac{i}{2r}\frac{1+\cos\vartheta}{\sin\vartheta}$

Bogomol'nyi eq. (G=U(1), k=1): 1 × 1 PDE

$$B_i = \partial_i \Phi$$

Φ :represents position of D3-brane

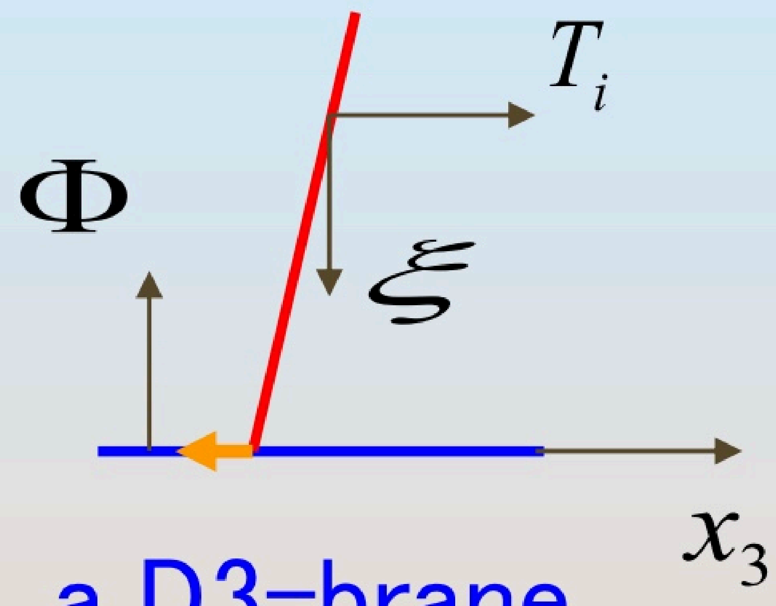
Nahm construction of NC Dirac monopoles

Gross&Nekrasov, JHEP[hep-th/0005204]

Nahm eq. (G=``U(1)’’): 1 × 1 ODE

$$\frac{dT_i}{d\xi} = i\varepsilon_{ijk}[T_j, T_k] - \theta\delta_{i3}$$

a D1-brane



a D3-brane

Nahm data $T_{1,2} = 0,\ T_3 = -\theta\xi$

1:1

monopoles $\Phi = \text{“}\frac{1}{2r} + \frac{x_3}{\theta}\exp\left(-\frac{x_1^2 + x_2^2}{\theta}\right)\text{“},\ A_i = \text{(smooth config.)}$

Bogomol’nyi eq. (G=U(1), k=1): 1 × 1 PDE

$$B_i = \partial_i \Phi$$

Φ :represents position of D3-brane

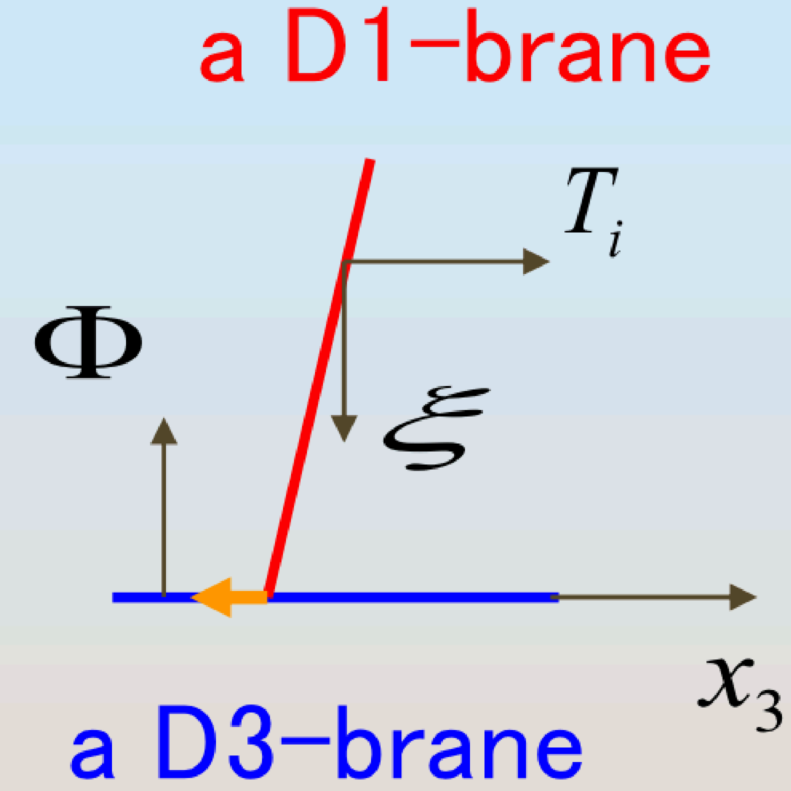
T_i :represents positions of D1-brane

Nahm construction of NC Dirac monopoles

Gross&Nekrasov, JHEP[hep-th/0005204]

Nahm eq. (G=``U(1)''): 1 × 1 ODE

$$\frac{dT_i}{d\xi} = i\varepsilon_{ijk}[T_j, T_k] - \theta\delta_{i3}$$



Nahm data $T_{1,2} = 0,\ T_3 = -\theta\xi$

1:1

Slope of D1-brane ⟷ Magnetic field pulls the end point of D1-brane

monopoles $\Phi = \text{“}\frac{1}{2r} + \frac{x_3}{\theta}\exp\left(-\frac{x_1^2 + x_2^2}{\theta}\right)\text{“},\ A_i =$ (smooth config.)

Bogomol'nyi eq. (G=U(1), k=1): 1 × 1 PDE

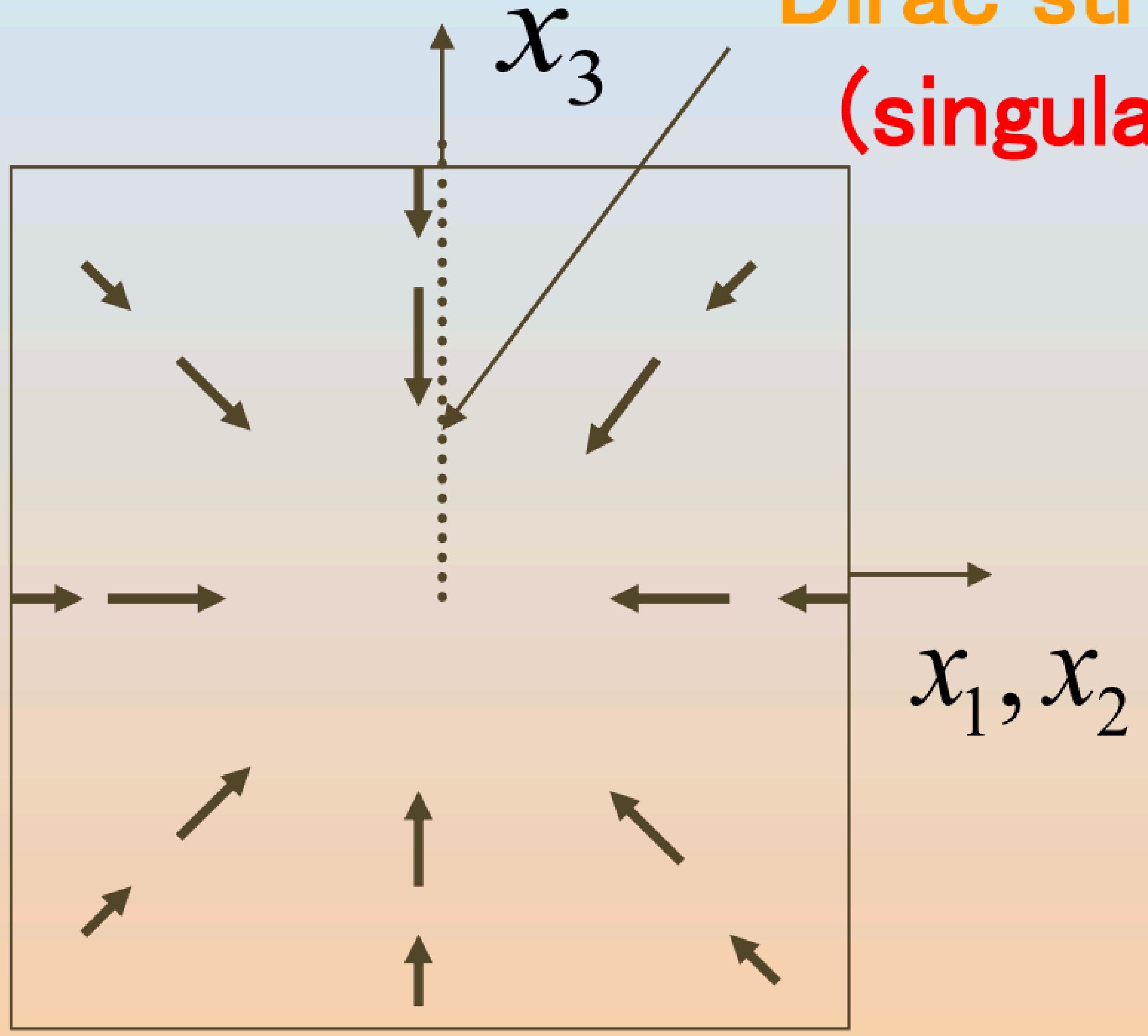
$$B_i = \partial_i \Phi$$

Φ :represents position of D3-brane

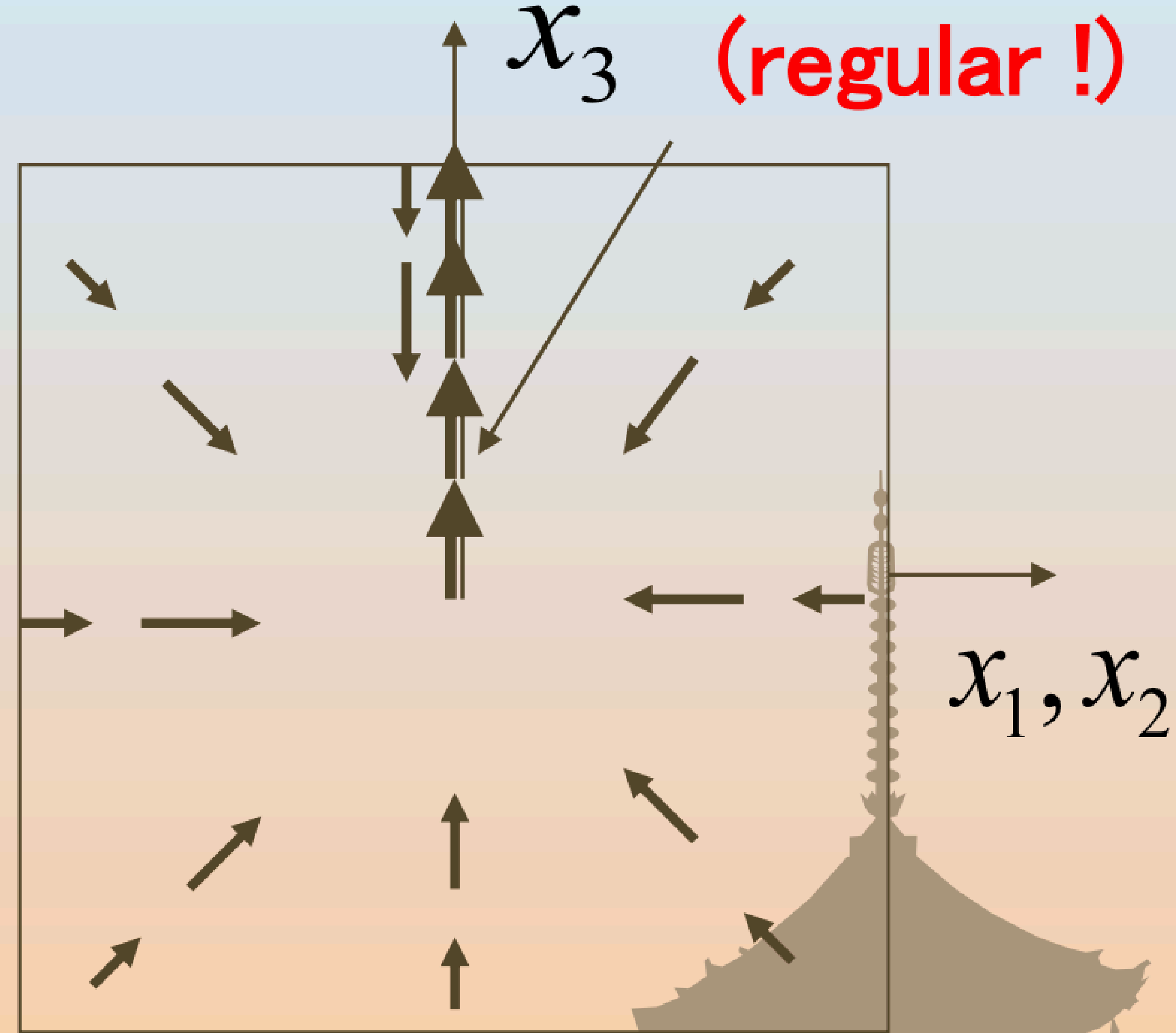
T_i :represents positions of D1-brane

Magnetic flux of Dirac monopoles

$$B_3 = \partial_3 \Phi = \frac{-x_3}{2r^3} + \frac{2}{\theta}\exp\left(-\frac{x_1^2 + x_2^2}{\theta}\right)$$



On commutative space



On NC space (roughly)

Solutions show interesting behaviors, though the moduli space is the same as commutative one.

4. Conclusion and Discussion

- We constructed instantons, global solutions of Anti-Self-Dual (ASD) Yang-Mills eqs.
- Extension to NC spaces corresponds to presence of background magnetic fields.

  →Resolution of singularity

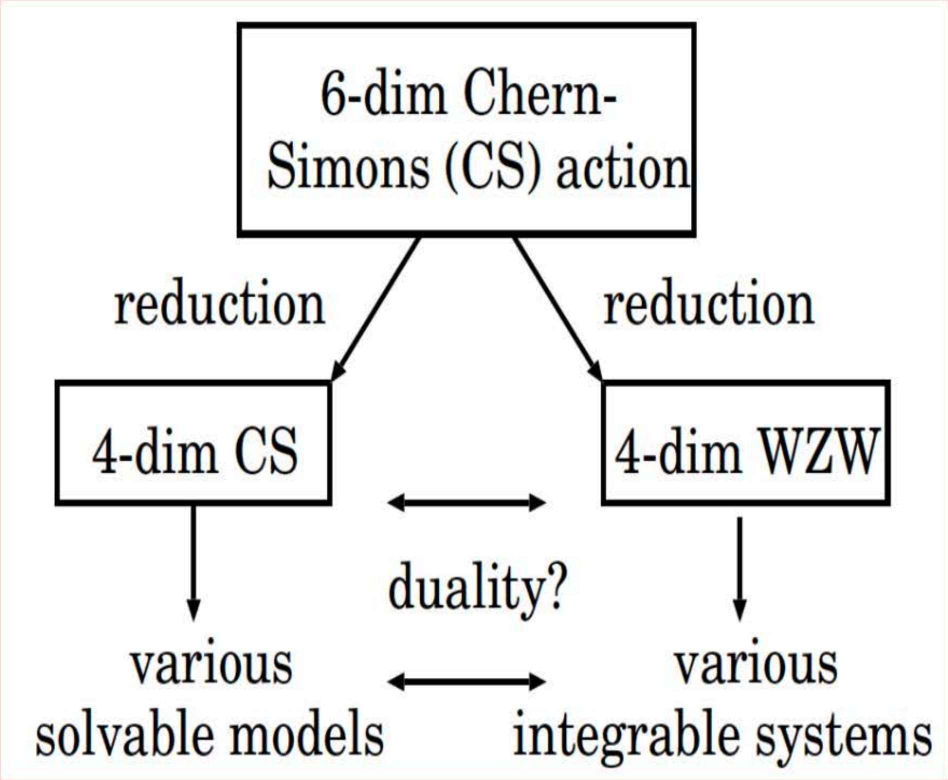
  →new physical objects (U(1) instantons,…)
- Local solutions (KP-type soliton solutions) of ASDYM eqs. are also interesting (relates to integrable systems). Recent works:[MH, Shan-Chi Huang, Hiroaki Kanno and Shangshuai Li,-Changzheng Qu, Xiangxuan Yi, Da-Jun Zhang, …]

Nagoya IAR workshop on

Unification of Integrable Systems

27(Mon)~29(Wed) October, 2025@Nagoya University

Speakers: B.Vicedo, M.Yamazaki, K.Yoshida and M.Hamanaka,...



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